UNIT-9 KINETIC THEORY OF GASES

MY REVISION TIMELINE:-

SUMMARY:-

- Kinetic theory explains the microscopic origin of macroscopic parameters like temperature, pressure.
- The pressure P = $\frac{1}{3}nm\overline{V^2}$. The pressure is directly proportional to the number density mass of molecules and mean square speed.
- The pressure is also equal to $\frac{2}{3}$ of internal energy per unit volume.
- Among the speeds V_{rms} is the largest and V_{mp} is the least. $V_{rms} > \bar{V} > V_{mp}$.
- For a monoatomic molecule, f = 3
- For a diatomic molecule, f = 5 (at normal temperature)
- For a diatomic molecule, f = 7 (at high temperature)
- For a triatomic molecule, f = 7 (linear type)
- For a triatomic molecule, f = 6 (non-linear type)
- The average kinetic energy of sample of gas is equally distributed to all the degrees of freedom. It is called law of equipartition of energy. Each degree of freedom will get ½ kT energy.
- The ratio of molar specific heat at constant pressure and constant volume of gas $\left[\gamma = \frac{C_p}{C_v}\right]$
- The mean free path $\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$. The mean free path is directly proportional to temperature and inversely proportional to size of the molecules and pressure of the molecules.
- The Brownian motion explained by Albert Einstein is based on kinetic theory. It proves the reality of atoms and molecules.

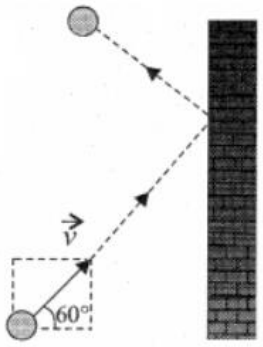
FORMULAS:-

- The force exerted by the molecules on the wall F = nmAV_x^2
- Pressure P = $\frac{1}{3}nm\overline{V^2}$ or P = $nm\overline{V_x^2}$
- Average kinetic energy KE = $\in$ = $\frac{3}{2}$kT
- Internal energy of an ideal gas U = $\frac{3}{2}$NkT
- Relation between pressure and mean kinetic energy P = $\frac{2}{3}\overline{kE}$
- Boyle's law PV = constant
- Charles's law $\frac{V}{T}$ = constant
- Root mean square speed v_{rms} = $\sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$

- Mean or average speed $\bar{v} = 1.60\sqrt{\frac{kT}{m}}$
- Most probable speed $v_{mp} = 1.41\sqrt{\frac{kT}{m}}$
- Maxwell-Boltzmann speed distribution $N_v = 4\pi N\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{\frac{-mv^2}{2kT}}$
- Mean free path $\lambda = \frac{1}{\sqrt{2}n\pi d^2}$

MULTIPLE CHOICE QUESTIONS:-

1. **A particle of mass m is moving with speed u in a direction which makes 60° with respect to x axis. It undergoes elastic collision with the wall. What is the change in momentum in x and y direction?**



a) ΔP_x = -mu, ΔP_y = 0

Solution:

As it moves with respect to x axis ΔPx = -mu, ΔPy = 0

2. **A sample of ideal gas is at equilibrium. Which of the following quantity is zero?**

c) average velocity

Solution:

$v_{mv} = 1.60\sqrt{\frac{kT}{m}}$

In equilibrium T = 0

3. **An ideal gas is maintained at constant pressure. If the temperature of an ideal gas increases from 100K to 1000K then the rms speed of the gas molecules**

b) increases by 10 times

Solution:

$v_{rms} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$

ΔT increases by 10 times

v_{rms} is directly proportional to ΔT

RMS speed increases by 10 times

4. Two identically sized rooms A and B are connected by an open door. If room A is air-conditioned such that its temperature is 4° lesser than room B, which room has more air in it?

a) Room A

Solution:

As temperature of room A is less than that of room B, evidently room A has more air in it.

5. The average translational kinetic energy of gas molecules depends on

a) number of moles and T

Solution:

Average KE of each degree of freedom = ½ kT = ½ NT (k=N)

N = no. of moles

T = temperature

6. If the internal energy of an ideal gas U and volume V are doubled then the pressure

b) remains same

Solution:

Pressure is independent of internal energy

7. The ratio $\gamma = \frac{C_p}{C_v}$ for a gas mixture consisting of 8 g of helium and 16 g of oxygen is

c) $\frac{27}{17}$

Solution:

$\gamma = \frac{27}{17}$

No. of moles of helium $n = \frac{8}{4} = 2$

No. of moles of oxygen $n' = \frac{16}{32} = = ½$

For monoatomic Helium gas f = 3

$C_v = \frac{f}{2}R = \frac{3}{2}R$

For diatomic Oxygen gas f = 5

$C_v = \frac{f}{2}R = \frac{5}{2}R$

C_v *mixture* $= \frac{nC_v + n'C'_v}{n+n'}$

$= \frac{2\times\frac{3}{2}R + \frac{1}{2}\times\frac{5}{2}R}{2+\frac{1}{2}}$

$= \frac{3R+\frac{5}{4}R}{\frac{5}{2}}$

$= \frac{17R}{10}$

$\gamma = \frac{C_p}{C_v} = 1 + \frac{R}{C_v} = 1 + \frac{R}{\frac{17R}{10}}$

$\gamma = 1 + \frac{10}{17}$

$$\boldsymbol{\gamma = \frac{27}{17}}$$

8. A container has one mole of a monoatomic ideal gas. Each molecule has /degrees of freedom. What is the ratio of $\gamma = \frac{C_p}{C_v}$?

d) $\frac{f+2}{f}$

Solution:

$V = \frac{f}{2}RT \quad C_v = \frac{f}{2}R$

$C_p = \left(1 + \frac{f}{2}\right)R$

$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

$$\boldsymbol{\gamma = \frac{f+2}{f}}$$

9. If the temperature and pressure of a gas is doubled the mean free path of the gas molecules

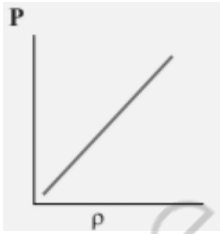
a) remains same

Solution:

Mean free path is independent of temperature and pressure.

10. Which of the following shows the correct relationship between the pressure and density of an ideal gas at constant temperature?

d)



Solution:

Pressure is directly proportional to density.

11. A sample of gas consists of μ_1 moles of monoatomic molecules, μ_2 moles of diatomic molecules, and μ_3 moles of linear triatomic molecules. The gas is kept at a high temperature. What is the total number of degrees of freedom?

a) $[3\mu_1 + 7(\mu_2 + \mu_3)]\, N_A$

Solution:

For monoatomic molecules, no of degree of freedom = 3

For diatomic molecules, no of degree of freedom = 5

For triatomic molecules, no of degree of freedom = 7

Total = $[3\mu_1 + 7(\mu_2 + \mu_3)]\, N_A$

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12. If S_P and S_V denote the specific heats of nitrogen gas per unit mass at constant pressure and constant volume respectively, then

b) $S_p - S_v = \frac{R}{28}$

Solution:

$C_p - C_v = R$

For diatomic gas (N_2), no of degrees of freedom = 5

$\therefore S_p - S_v = \frac{R}{28}$

13. Which of the following gases will have the least RMS speed at a given temperature?

d) Carbon-dioxide

Solution:

Carbon-dioxide

$$v_{rms} = 1.73\sqrt{\frac{kT}{m}}$$

14. For a given gas molecule at a fixed temperature, the area under the Maxwell-Boltzmann distribution curve is equal to

a) $\frac{PV}{kT}$

Solution:

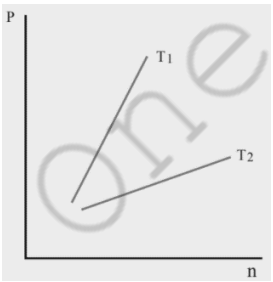
The area under the graph will give total number of gas molecules in the system.

$n = \frac{PV}{RT}$

$R = k$

$$\therefore n = \frac{PV}{kT}$$

15. The following graph represents the pressure versus number density for an ideal gas at two different temperatures T_1 and T_2. The graph implies



b) $T_1 > T_2$

Solution:

From the graph, we get $T_1 > T_2$

NUMERICAL PROBLEMS:-

1. Given:

T = 20°C

Formula used:

$v_{rms} = \sqrt{\frac{3RT}{m}}$

Solution:

For N_2, molar mass = 0.0280 kg/mol

T = 293K

R = 8.314 J/mol K

$v_{rms} = \sqrt{\frac{3RT}{m}}$

$v_{rms} = \sqrt{\frac{3\times8.314\times293}{0.0280}}$

$v_{rms} = \sqrt{2610 \times 10^2}$

v_{rms} = 511 m/s

For O_2, molar mass = 0.0320 kg/mol

T = 293K

R = 8.314 J/mol K

$v_{rms} = \sqrt{\frac{3RT}{m}}$

$v_{rms} = \sqrt{\frac{3\times8.314\times293}{0.0320}}$

$v_{rms} = \sqrt{2280 \times 10^2}$

v_{rms} = 478 m/s

2. **Given:**

RMS speed of methane gas = 471.8 m/s = v_{rms}

Molar mass of the methane gas = 16.04 × 10^{-3} kg/mole

Surface temperature of Jupiter = ?

Formula used:

$v_{rms} = \sqrt{\frac{3RT}{m}}$

$T = \frac{v_{rms}^2 M}{3R}$

Solution:

$T = \frac{v_{rms}^2 M}{3R}$

$T = \frac{(471.8)^2\times16.04\times10^{-3}}{3\times8.31}$

$T = \frac{3.549456\times10^{-3}}{24.9}$

T = 143 K or -130°C

3. **Given:**

Velocity of the gas triples

T_1 = 273K

T_2 =?

Formula used:

$v_{rms} = \sqrt{\frac{3RT}{m}}$

Solution:

v_{rms} of gas v_{rms_1} = v

RMS velocity of gas triples i.e. $v_{rms_2} = 3v$
New temperature at v_{rms_2} *= ?*

$$\frac{v_{rms_1}^2}{v_{rms_2}^2} = \frac{T_1}{T_2}$$

$$\frac{v^2}{(3v)^2} = \frac{273}{T_2}$$

$T_2 = 273\ (9)$

$T_2 = 2457K$

4. Given:

T = 80°C = 353K
Pressure P = 5 × 10^{-10} Nm^{-2}
Boltzmann's constant = 1.38 × 10^{-23} JK^{-1}
V = 1m^3
No. of molecules n = ?

Formula used:

$PV = NkT$

$$N = \frac{PV}{kT}$$

Solution:

$$N = \frac{PV}{kT}$$

$$N = \frac{5\times10^{-10}\times1}{1.38\times10^{-23}\times353}$$

$$N = \frac{5\times10^{-10}}{480\times10^{-23}}$$

$N = 0.01026 \times 10^{13}$

$N = 1.02 \times 10^{11}$

5. Given:

Mass of one atom = 2.67 × 10^{-26}
Mass of O_2 atom = 16 × 10^{20} × 2.67 × 10^{-26} = 42.72 × 10^{-6}
Area = 4cm^2
Angle = 30°
Velocity = 2 × 10^3 m/s

Formula used:

$$Pressure = \frac{F}{A} = \frac{Change\ in\ momentum}{Area}$$

Solution:

Momentum of O_2 molecule (p)= mv = 42.72 × 10^{-6} × 2 × 10^3 = 85.44 × 10^{-3} kgms^{-1}
Momentum normal to the wall at 30° = 85.44 × 10^{-3} × cos 30° = 73.99 × 10^{-3} kg ms^{-1}

$$Pressure = \frac{F}{A} = \frac{Change\ in\ momentum}{Area}$$

$$Pressure = \frac{73.99\times10^{-3}}{(4\times10^{-2})^2}$$

$$Pressure = \frac{73.99\times10^{-3}\times10^4}{16}$$

$Pressure = 4.62 \times 10$

Pressure = 46.2 Nm^{-2}

6. Solution:

$P^{1-\gamma}T^{\gamma} = constant$
$P\ \alpha\ T^3$
$[T^3]^{1-\gamma}T^{\gamma} = constant$

$T^{3-3\gamma}T^{\gamma} = constant$
$T^{3-2\gamma} = constant$
$3 - 2\gamma = 0$
$3 = 2\gamma$

$$\boxed{\gamma = \frac{3}{2}}$$

7. Given:
$d = 3 \times 10^{-10}m$
Formula used:
$n = \frac{N}{V} = \frac{P}{kT}$
$\lambda = \frac{1}{\sqrt{2}n\pi d^2}$
Solution:
$n = \frac{P}{kT}$
$n = \frac{101.3\times10^3}{1.381\times10^{-23}\times273}$
$n = 0.2687 \times 10^{26}$
$\lambda = \frac{1}{\sqrt{2}n\pi d^2}$
$\lambda = \frac{1}{1.414\times3.14\times0.2687\times10^{26}\times(3\times10^{-2})^2}$
$\lambda = \frac{1}{10.74\times10^6}$

$$\boxed{\lambda = 9.3 \times 10^{-8}m}$$

8. Formula used:
$V = \frac{n_1 f_1}{2}RT + \frac{n_2 f_2}{2}RT$ *where f is number of degree of freedom*
Solution:
f_1 *- for diatomic gas = 5*
f_2 *- for monoatomic gas = 3*
$V = \frac{n_1 f_1}{2}RT + \frac{n_2 f_2}{2}RT$
$V = \frac{2\times5}{2}RT + \frac{4\times3}{2}RT$
$V = 5RT + 6RT$

$$\boxed{V = 11RT}$$

9. Given:
Volume of room $V = 25m^3$
$T = 27°C = 300K$
$P = 1.013 \times 10^5 Pm$
Formula used:
$PV = k_B NT$
$N = \frac{PV}{k_B T}$
Solution:
$N = \frac{PV}{k_B T}$
$N = \frac{1.013\times10^5\times25}{1.38\times10^{-23}\times300}$

$$\boxed{N = 6.1 \times 10^{26} molecules}$$