UNIT-6 GRAVITATION

MY REVISION TIMELINE:-

SUMMARY:-

- The motion of planets can be explained using Kepler's law.
- KEPLER'S FIRST LAW: All the planets in the solar system orbit the Sun in elliptical orbits with the Sun at one of the foci.
- KEPLER'S SECOND LAW: The radial vector line joining the Sun to a planet sweeps equal areas in equal intervals of time.
- KEPLER'S THIRD LAW: The ratio of the square of the time period of planet to the cubic power of semi major axis is constant for all the planets in the solar system.
- NEWTON'S LAW OF GRAVITATION: It states that the gravitational force between two masses is directly proportional to product of masses and inversely proportional to square of the distance between the masses. In vector form it is given by $\vec{F} = \frac{-Gm_1m_2}{r^2}\hat{r}$
- Gravitational force is a central force.
- Kepler's law can be derived from Newton's law of gravitation.
- The gravitational field due to a mass m at a point which is at a distance r from mass m is given by $\vec{E} = \frac{-Gm}{r^2}\hat{r}$. It is a vector quantity.
- The Gravitational potential energy of two masses is given by U = $\frac{-Gm_1m_2}{r^2}$. It is a scalar quantity.
- The Gravitational potential at a point which is at a distance r from mass m is given by V = $\frac{-Gm}{r}$. It is a scalar quantity.
- The acceleration due to Earth's gravity decreases as altitude increases and depth increases.
- Due to rotation of the Earth, the acceleration due to gravity is maximum at poles and minimum at equator.
- The (escape) speed of any object required to escape from the Earth's gravitational field is $v_e = \sqrt{2gR_e}$. It is independent of mass of the object.
- The energy of the satellite is negative. It implies that the satellite is bound to Earth's gravitational force.
- Copernicus model explained that retrograde motion is due to relative motion between planets. This explanation is simpler than Ptolemy's epicycle explanation which is complicated.
- Copernicus and Kepler measured the distance between a planet and the Sun using simple geometry and trigonometry.
- 2400 years ago, Eratosthenes measured the radius of the Earth using simple geometry and trigonometry.

FORMULAS:-

- Universal law of gravitation: $\vec{F} = \frac{-Gm_1m_2}{r^2}\hat{r}$
- Newton's inverse square law: $F = \frac{-GMm}{r^2}$
- Superposition principle for Gravitational field: $\overrightarrow{E_{total}} = \sum_{i=1}^{n} \frac{Gm}{r_i^2}\hat{r_i}$
- The total work done for displacing the particle from r′ to r is W = U(r) – U(r′) where U(r) = $\frac{-Gm_1m_2}{r}$
- The potential energy difference between h_1 and h_2 is $U(h_2) - U(h_1) = mg(h_2 - h_1)$
- The gravitational potential V(r) at a point of height h from the surface of the Earth is given by V(r = R+h) = $\frac{-GM_e}{R+h}$
- The gravitational potential V(r) on the surface of the Earth is given by V(r = R) = $\frac{-GM_e}{R}$
- ∴ V(r = R) < V(r = R+h)
- The magnitude of acceleration due to gravity is |g| = $\frac{GM_e}{R_e^2}$
- Variation of 'g' with altitude g′ = g $\left[1 - \frac{2h}{R_e}\right]$ g = $\frac{GM}{R_e}$, g′ < g
- Variation of 'g' with depth g′ = g $\left[1 - \frac{d}{R_e}\right]$ g′ < g
- Variation of 'g' with latitude g′ = g - $\omega^2 R\cos^2\lambda$
- Escape speed, $v_e = \sqrt{2gR_e}$
- Speed of the satellite, $v^2 = \sqrt{\frac{GM_E}{R_E+h}}$
- Time period of the satellite, $T = 2\pi\sqrt{\frac{R_E}{g}}$
- Energy of an orbiting satellite E = $\frac{-GM_SM_E}{2(R_E+h)}$

MULTIPLE CHOICE QUESTIONS:-

1. The linear momentum and position vector of the planet is perpendicular to each other at

a) perihelion and aphelion

2. If the masses of the Earth and Sun suddenly double, the gravitational force between them will

c) increase 4 times

Solution:

Gravitation Force F α m_1m_2

If $m_1 = 2m_1$ and $m_2 = 2m_2$ then

F α $(2m_1)(2m_2)$

F α $4m_1m_2$

3. **A planet moving along an elliptical orbit is closest to the Sun at distance r_1 and farthest away at a distance of r_2. If v_1 and v_2 are linear speeds at these points respectively. Then the ratio $\frac{v_1}{v_2}$ is [NEET 2016]**

   a) $\frac{r_2}{r_1}$

4. **The time period of a satellite orbiting Earth in a circular orbit is independent of**

   b) The mass of the satellite

   Solution:

   Time period $T = 2\pi\sqrt{\frac{R_E+h}{GM_E}}$

   ∴ It is independent of ***mass.***

5. **If the distance between the Earth and Sun were to be doubled from its present value, the number of days in a year would be**

   b) 1032

   Solution:

   $T^2 = c(R_E+h)^3$

   $T \alpha R_E^{3/2}$

   $R_E = 2R_E$

   $\therefore T \alpha 2R_E^{3/2}$

   Time period increases by $2^{3/2} = 2\sqrt{2}$

   ∴ No. of days in a year = $(365.4)\ 2\sqrt{2}$

   =1032

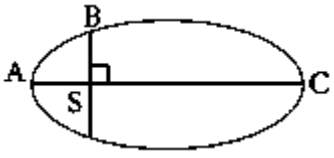
6. **According to Kepler's second law, the radial vector to a planet from the Sun sweeps out equal areas in equal intervals of time. This law is a consequence of**

   b) conservation of angular momentum

7. **The gravitational potential energy of the Moon with respect to Earth is**

   b) always negative

8. **The kinetic energies of a planet in an elliptical orbit about the Sun, at positions A, B and C are K_A, K_B and K_C respectively. AC is the major axis and SB is perpendicular to AC at the position of the Sun S as shown in the figure. Then [NEET 2018]**



   a) $K_A > K_B > K_C$

9. **The work done by the Sun's gravitational force on the Earth is**

   c) can be positive or negative

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10. If the mass and radius of the Earth are both doubled, then the acceleration due to gravity g´

b) $\frac{g}{2}$

Solution:

$g = \frac{GM_e}{R_e^2}$

$M_e = 2M_e$ $R_e = 2R_e$ then

$g' = \frac{G \times 2M_e}{(2R_e)^2}$

$g' = \frac{2GM_e}{4R_e^2}$

$g' = \frac{GM_e}{2R_e^2}$

$\boxed{g' = \frac{g}{2}}$

11. The magnitude of the Sun's gravitational field as experienced by Earth is

c) decreases in the month of July and increases in the month of January

12. If a person moves from Chennai to Trichy, his weight

b) decreases

13. An object of mass 10 kg is hanging on a spring scale which is attached to the roof of a lift. If the lift is in free fall, the reading in the spring scale is

b) zero

14. If the acceleration due to gravity becomes 4 times its original value, then escape speed

b) 2 times of original value

Solution:

Escape speed, $v_e = \sqrt{2gR_e}$

If $g' = 4g$

Then $v_e' = \sqrt{2(4g)R_e}$

$= 2\sqrt{2gR_e}$

$\boxed{= 2v_e}$

15. The kinetic energy of the satellite orbiting around the Earth is

b) less than potential energy

NUMERICAL PROBLEMS:-

1. Given:

Distance of an unknown planet is twice that of the Earth.

Time period of an unknown planet = ?

Formula used:

Time Period, $T = 2\pi\sqrt{\frac{R_E}{g}}$

Solution:

Let the distance of the Earth be R_E

The distance of unknown planet = $R_P = 2R_E$

Let the time period of the Earth be T_1

The time period of an unknown planet = T_2 = ?

$T = 2\pi\sqrt{\frac{R_E}{g}}$

$T \alpha \sqrt{R_E}$

$\frac{T_1}{T_2} = \sqrt{\frac{R_E}{R_P}}$

$\frac{T_1}{T_2} = \sqrt{\frac{R_E}{2R_E}}$

$\frac{T_1}{T_2} = \sqrt{\frac{1}{2}}$

$$\boxed{T_2 = \sqrt{2}T_1}$$

2. Solution:

The value of semi major axis is directly proportional to twice the square of time period of a planet. i.e., a α $2T^2$

It is given that for planet Kurinji, $T_1 = 2$, $a_1 = 8 = 2 \times 2^2$ => $\mathbf{2T_1^2}$

For planet Mullai, $T_2 = 3$, $a_2 = 18 = 2 \times 3^2$ => $\mathbf{2T_2^2}$

For planet Marutham, $T_3 = 4$, $a_3 = 32 = 2 \times 4^2$ => $\mathbf{2T_3^2}$

For planet Neithal, $T_4 = 5$, $a_4 = 50 = 2 \times 5^2$ => $\mathbf{2T_4^2}$

For planet Paalai, $T_5 = 6$, $a_5 = 72 = 2 \times 6^2$ => $\mathbf{2T_5^2}$

$$\therefore \mathbf{a \alpha 2T^2}$$

3. Solution:

Gravitational force $\vec{F} = \frac{-Gm_1m_2}{r^2}\hat{r}$

If masses and mutual distance between the two objects are doubled, there will be ***no change*** *in the gravitational force between them.*

4. Given:

Two masses m and 4m

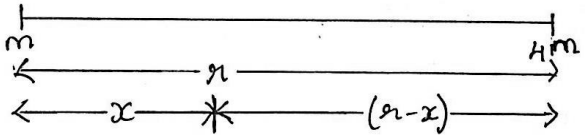
Distance = r

Gravitational field = 0

Formula used:

Gravitational field = $\frac{-Gm}{r^2}\hat{r}$

Solution:



$\frac{Gm}{x^2} = \frac{G \times 4m}{(r-x)^2}$

$\frac{1}{x^2} = \frac{4}{(r-x)^2}$

By taking square root on both the sides,

$\frac{1}{x} = \frac{2}{r-x}$

$r\text{-}x = 2x$
$r = 3x$
$x = \frac{r}{3}$ *for mass m*
$x = \frac{2r}{3}$ *for mass 4m*
Gravitational Potential $V = \frac{-Gm}{\frac{r}{3}} - \frac{G(4m)}{\frac{2r}{3}}$

$$\boldsymbol{V = \frac{-9Gm}{r}}$$

5. Given:
Ratio of orbital distance $\frac{d_1}{d_2} = 2$
Formula used:
Gravitational field $E_1 = \frac{-GM}{r^2}\hat{r}$
Solution:
$E_1 = \frac{-GM}{d_1^2}$, $E_2 = \frac{-GM}{d_2^2}$
$\frac{E_1}{E_2} = \frac{-GM}{d_1^2} \times \frac{-d_2^2}{GM}$
$\frac{E_1}{E_2} = \left[\frac{d_2}{d_1}\right]^2 = \left[\frac{1}{2}\right]^2 = \frac{1}{4}$

$$\boldsymbol{E_2 = 4E_1}$$

6. Given:
Time period = 1.769 days
Orbital radius = 421700 km = 421700000 m
Formula used:
$T = 2\pi\sqrt{\frac{R^3}{GM}}$
Squaring on both sides $T^2 = \frac{4\pi^2 R^3}{GM}$
$T^2 \alpha R^3$
Solution:
$\frac{R^3}{T^2} = \left[\frac{421700\times10^3}{1.769}\right]^{3/2}$
$\frac{R^3}{T^2} = \left[\frac{421.7\times10^6}{(1.769)^2}\right]^{3/2}$
$\frac{R^3}{T^2} = \left[\frac{7499.14\times10^{22}}{3.1293}\right]^{3/2}$
$\frac{R^3}{T^2} = 2396.4 \times 10^{22}$
$M = 4 \times 3.14 \times 6.67 \times 10^{-11} \times 2396.4 \times 10^{22}$

$$\boldsymbol{M = 1.898 \times 10^{27}\ kg}$$

7. Given:
Angular momentum $L = 5t^2\hat{\imath} - 6t\hat{\jmath} + 3\hat{k}$
Formula used:
Torque $\alpha \frac{dL}{dt}$
Solution:
Torque $\alpha \frac{dL}{dt}$

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Torque $= \frac{d}{dt}[5t^2\hat{\imath} - 6t\hat{\jmath} + 3\hat{k}]$

$\boxed{\textbf{Torque} = 10t\hat{\imath} - 6\hat{\jmath}}$

8. Solution:

The gravitational potential energy $V = \frac{-GM^2}{R}\left[1 + \frac{4}{\sqrt{2}}\right] = \frac{-GM^2}{R}\left[1 + 2\sqrt{2}\right]$

Gravitational potential $V_o(r) = \frac{-4GM}{R}$

Centripetal acceleration $a = \frac{v^2}{R}$

Centripetal force $= \frac{Mv^2}{R}$

$\boxed{\textbf{Speed } v = \frac{1}{2}\sqrt{\frac{GM}{R}\left[1 + 2\sqrt{2}\right]}}$

9. Given:

Incorrect value $G = 6.67 \times 10^{11}\ Nm^2kg^{-2}$

Radius of the earth $R_e = 6400\ km = 6.4 \times 10^6 m$

Mass of earth $M_e = 6.02 \times 10^{24} kg$

Formula used:

Acceleration due to gravity $g = \frac{GM_e}{R_e^2}$

$W = mg$

Solution:

$g' = \frac{GM_e}{R_e^2}$

$g' = \frac{6.67\times10^{11}\times6.02\times10^{24}}{(6.4\times10^6)^2}$

$g' = \frac{40.1534}{40.96}\ 10^{23}$

$g' = 0.9830 \times 10^{23}$

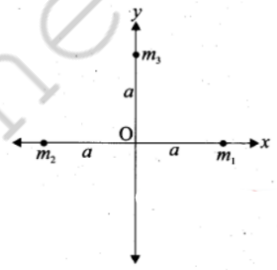
$\boxed{\textbf{g}' = 10^{22}}$

$W = mg$

$W' = mg'$

$\boxed{\textbf{W}' = 10^{22}\textbf{W}}$

10. Solution:



From the figure, the distance of m_1 *and* m_2 *from the origin* $= a$

Gravitational field $E = \frac{-GM}{r^2}\hat{r}$

At the origin (point O) the change in gravitational field is $\vec{E} = \frac{GM}{a^2}[(m_1 - m_2)i + m_3\hat{\jmath}]$

It is given that $m_1 = m_2$

$\boxed{\overrightarrow{\boldsymbol{E_1}} = \frac{\boldsymbol{GM}}{\boldsymbol{a^2}}[\boldsymbol{m_3\hat{\jmath}}]}$

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11. Given:

Mass of the Earth $M_E = 5.9 \times 10^{24}$ *kg*

Mass of the Sun $M_S = 1.9 \times 10^{30}$ *kg*

Distance between the Sun and Earth R = 150 million km $= 150 \times 10^9$ *m*

Gravitational constant $G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$

Formula used:

$U = \frac{-GM_EM_S}{R}$

Solution:

$U = \frac{-GM_EM_S}{R}$

$U = -\frac{6.67\times10^{-11}\times5.9\times10^{24}\times1.9\times10^{30}}{150\times10^9}$

$U = -0.49847 \times 10^{34}$

$$\boxed{U = -49.847 \times 10^{32} J}$$

12. Given:

Velocity of the Earth around Sun v = 30km/s

Formula used:

Kinetic energy K.E. $= \frac{-GM_SM_E}{2(R_E+h)}$

Solution:

$K.E. = \frac{M_Ev^2}{2} = \frac{1.9\times10^{30}\times(30\times10^3)^2}{2} = 26.5 \times 10^{32} J\ (U_k)$

Total energy $E = \frac{-GM_SM_E}{2(R_E+h)}$

$E = \frac{-6.67\times10^{-11}\times1.9\times10^{36}\times59\times10^{24}}{2\times6.4\times10^6}$

$$\boxed{E = -23.29 \times 10^{32} J}$$

Total energy has the negative sign. The negative sign in the total energy implies that the satellite is bound to earth and earth cannot escape from the Sun i.e. –ve sign implies that earth is bounded with Sun.

13. Given:

At Earth's surface KE $= \frac{1}{2}Mv_e^2$, *PE* $= \frac{-GMM_E}{R_E}$

At infinity KE $= \frac{1}{2}Mv_\infty^2$, *PE = 0*

Solution:

By law of conservation of energy

$\frac{1}{2}Mv_e^2 - \frac{-GMM_E}{R_E} = \frac{1}{2}Mv_\infty^2$

$v_e^2 = v_\infty^2 + \frac{2GM_E}{R_E}$

But $GM_E = qR_E^2$

$v_e^2 = v_\infty^2 + 2gR_E$

$$\boxed{v_e = \sqrt{v_\infty^2 + 2gR_E}}$$

14. Given:

Height = Altitude = h = 200km

Depth d = 200km

$R_e = 6400km$

Formula used:

$G_{up} = g\left[1 - \frac{2h}{R_e}\right]$

$g_{down} = g\left[1 - \frac{d}{R_e}\right]$

Solution:

$g_{up} = g\left[1 - \frac{2h}{R_e}\right]$

$g_{up} = g\left[1 - \frac{2(200)}{6400}\right]$

$g_{up} = g\left[\frac{6400-400}{6400}\right]$

$g_{up} = g\left[\frac{6000}{6400}\right]$

$\boxed{g_{up} = 0.94\ g}$

$g_{down} = g\left[1 - \frac{d}{R_e}\right]$

$g_{down} = g\left[1 - \frac{200}{6400}\right]$

$g_{down} = g\left[\frac{6400-200}{6400}\right]$

$g_{down} = g\left[\frac{6200}{6400}\right]$

$\boxed{g_{down} = 0.96\ g}$

*The value of g is small **200km above the surface surface of the Earth**.*

15. Formula used:

$g_{latitude}\ g' = g - \omega^2 R\cos^2\lambda$

Solution:

Value of latitude of g at Chennai = 13°

$\omega = \frac{2\pi}{T}$

$\omega = \frac{2\times3.14}{24\times3600}$

$\omega = \frac{2\times3.14}{86400}$

$\therefore \omega^2 R = \left[\frac{2\times3.14}{86400}\right]^2 \times 6400 \times 10^3$

$\omega^2 R = 3.4 \times 10^{-2}\ m/s^2$

$g_{Chennai} = g - \omega^2 R\cos^2\lambda$

$g_{Chennai} = 9.8 - (3.4 \times 10^{-2})^2 \cos(0.2268)^2$

$\boxed{g_{Chennai} = 9.7677\ m/s^2}$

Value of latitude of g at Kanyakumari = 8.088°N = 8.1°N

$\omega = \frac{2\pi}{T}$

$\omega = \frac{2\times3.14}{24\times3600}$

$\omega = \frac{2\times3.14}{86400}$

$\therefore \omega^2 R = \left[\frac{2\times3.14}{86400}\right]^2 \times 6400 \times 10^3$

$\omega^2 R = 3.4 \times 10^{2}\ m/s^2$

$g_{Kanyakumari} = g - \omega^2 R\cos^2\lambda$

$g_{Kanyakumari} = 9.8 - (3.4 \times 10^{2})^2 \cos(8.1)^2$

$\boxed{g_{Kanyakumari} = 9.798\ m/s^2}$

$\Delta g = 9.798 - 9.767$

$\boxed{\Delta g = 0.031\ m/s^2}$

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CONCEPTUAL QUESTIONS:-

1. **In the following, what are the quantities which are conserved?**
   b) Angular momentum of planet
2. **The work done by Sun on Earth in one year will be**
   d) negative
3. **The work done by Sun on Earth at any finite interval of time is**
   c) strictly positive
4. **If a comet suddenly hits the Moon and imparts energy which is more than the total energy of the Moon, what will happen?**
   Kinetic energy of the **Moon** will **vary** but its **Total energy** remains **constant.**

5. **If the Earth's pull on the Moon suddenly disappears, what will happen to the Moon?**
   Moon will start to move in **spiral path** towards the Earth and it may hit on the surface of the Earth.

6. **If the Earth has no tilt, what happens to the seasons of Earth?**
   If the Earth has no tilt, then the **seasons** of the Earth **may change** and existence of the life on Earth would be affected.

7. **A student was asked a question 'why are there summer and winter for us? He replied as 'since Earth is orbiting in an elliptical orbit, when the Earth is very far away from the Sun (aphelion) there will be winter, when the Earth is nearer to the Sun (perihelion) there will be winter}. Is this answer correct? If not, what is the correct explanation for the occurrence of summer and winter?**
   The answer is **wrong.**
   Actually, the seasons in the Earth arise due to the **rotation of the Earth** around the Sun with **23.5° tilt.**

8. **The following photographs are taken from the recent lunar eclipse which occurred on January 31, 2018. Is it possible to prove that Earth is a sphere from these photographs?**
   From the shadows it is revealed that Earth has **spherical shape** with **bulging** along the **equator** and **flat** at **poles**.

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