ONE TWO ACADEMY

Unit Test - 07

TOTAL:- 45 **GENERAL MATHEMATICS** **STD XII**

Application of Derivatives

Choose the correct answer:- ***5 x 1 = 5***

1. The number given by Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0,3]$ is …..

(a) 1 (b) $\sqrt{2}$ (c) $\frac{3}{2}$ (d) 2

2. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(a) $\tan^{-1}\frac{3}{4}$ (b) $\tan^{-1}\frac{4}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

3)The point of inflection of the curve $y = (x-1)^3$ is …….

(a) (0,0) (b) (0,1) (c) (1,0) (d) (1,1)

4)If the normal makes an angle θ with a positive x-axis then the slope of the curve at the point where the normal is drawn is ……

(a) $-\cot\theta$ (b) $\tan\theta$ (c) $-\tan\theta$ (d) $\cot\theta$

5)Which of the following curves in concave down?

(a) $y = -x^2$ (b) $y = x^2$ (c) $y = e^x$ (d) $y = x^2 + 2x - 3$

Answer any 5 of the following (Question no 8 is compulsory):- ***5 x 2 = 10***

6)The temperature T in celsius in a long rod of length 10 m, insulated at both ends, is a function of length x given by $T = x(10-x)$. Prove that the rate of change of temperature at the midpoint of the rod is zero.

7)Find the points on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is horizontal.

8)Find the asymptotes of the function $f(x) = \frac{1}{x}$.

9)Write the Maclaurin series expansion of the following functions: $f(x) = e^x$.

10)$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.

11)A thermometer was taken from a freezer and placed in boiling water. It took 22 seconds for the thermometer to raise from -10^o C to 100°C. Show that the rate of change of temperature at some time t is 5°C per second.

12)A particle moves along a straight line in such a way that after t seconds its distance from the origin is $s = 2t^2 + 3t$ meters

(i) Find the average velocity between t = 3 and t = 6 seconds.

(ii) Find the instantaneous velocities at t = 3 and t = 6 seconds.

13)If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when x = 5 units.

Answer any 5 of the following (Question no 21 is compulsory):- ***7 x 3 = 21***

14)Show that $x^2 - y^2 = a^2$ and $xy = c^2$ cuts orthogonally.

15)A particle moves along a line according to the law , where $t \geq 0$. Find the total distance travelled by the particle in the first 4 seconds.

16)A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5km from the straight shoreline. How fast is the beam moving along the shoreline when it makes an angle of 45^o with the shore?

17) Using the mean value theorem prove that for, a> 0 and b>0 $|e^{-a} - e^{-b}| < |a-b|$

18)Prove that $\lim_{x \to 0+} x^x = 1$.

19)Find the absolute extrema of the following function on the given closed interval.

$f(x) = x^2 - 12x + 10$; [1,2]

20)Find the two positive numbers whose product is 20 and their sum is minimum

21)Write down the Taylor series expansion, of the function log x about x =1 up to three non-zero terms for x > 0.

Answer the following:- **3 x 5 = 15**

22)Find the angle between $y = x^2$ and $y = (x-3)^2$.

[OR]

A ladder 17 meters long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5m/s. When the base of the ladder is 8 meters from the wall,

a)How fast is the top of the ladder moving down the wall?

b)At what rate, the area of the triangle formed by the ladder, wall and floor, is changing?

23)A farmer wants to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material?

[OR]

A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times the volume of the cone.

24)Sketch the curve $y = f(x) = x^2 - x - 6$.

[OR]

Expand tan x in ascending powers of x upto 5th power for $\frac{-\pi}{2} < x < \frac{\pi}{2}$.

All the best