One Two academy

STD 12 PHYSICS SLOW LEARNER'S MATERIAL

UNIT 1 ELECTROSTATICS

Two Mark questions

1. What is meant by the quantisation of electric charge?

The charge q on any object is equal to an integral multiple of this fundamental unit of charge(e).

2. Write down Coulomb's law in vector form and mention what each term represents

$$\overrightarrow{F_{21}} = k\frac{q_1q_2}{r^2}\hat{r_{12}}$$

The force on point charge q_2 exerted by another point charge q_1 is $\overrightarrow{F_{21}}$.

$\hat{r_{21}}$ is directed from q_1 to the charge q_2.

k is the proportionality constant.

3. What are the differences between Coulomb force and gravitational force?

Columb force	Gravitational force
Attractive or repulsive	Always attractive
Dependent on medium	Independent on medium
Comparatively greater force	Comparatively weaker force

4. Write a short note on the superposition principle.

The total force acting on a given charge is equal to the vector sum of forces exerted on it by all the other charges.

5.Define 'electric field'.

Force experienced by a unit positive charge.

6.What is meant by 'electric field lines'?

A set of continuous lines which are the visual representation of the electric field in some region of space.

7.The electric field lines never intersect. Justify.

If some charge is placed in the intersection point, then it has to move in two different directions at the same time, which is physically impossible..

8.Define 'electric dipole'. Give the expression for the magnitude of its electric dipole moment and the direction.

- Two equal and opposite charges separated by a small distance constitute an electric dipole.
- p = 2qa

- Direction: –q to +q.

9.Write the general definition of electric dipole moment for a collection of point charges.

$\vec{p} = \sum_{i=1}^{n} q_i \vec{r_i}$ $\vec{r_i}$ is the position vector of charge q ii from the origin.

10. Define 'electrostatic potential".

The electric potential at a point P is equal to the work done by an external force to bring a unit positive charge with constant velocity from infinity to the point P in the region of the external electric field $\vec{E}$

11. What is an equipotential surface?

An equipotential surface is a surface on which all the points are at the same electric potential.

12. What are the properties of an equipotential surface?

- The work done to move a charge in an equipotential surface is zero.
- The electric field must always be normal to the equipotential surface.

13. Give the relation between electric field and electric potential.

$$dW = dV \qquad dW = -E\,dx$$

$$dV = -E dx$$

$$E = -\frac{dV}{dx}$$

14. Define 'electrostatic potential energy'.

Work done in bringing a charge from an infinite distance.

15. Define 'electric flux'.

The number of electric field lines crossing a given area kept normal to the electric field lines is called electric flux.

16. What is meant by electrostatic energy density?

Energy stored per unit volume of space

17. Write a short note on 'electrostatic shielding'. What is polarisation?

Electrostatic shielding:

It is the process of isolating a certain region of space from an external field. It is based on the fact that the electric field inside a conductor is zero.

Polarisation:

Total dipole moment per unit volume of the dielectric

18. What is dielectric strength?

The maximum electric field the dielectric can withstand before it breaks down is called dielectric strength.

19. Define 'capacitance'. Give its unit.

$C = \frac{Q}{V}$ [Q - charge on any one side of conductor | V - Potential difference]

Unit: farad (F)

20. What is corona discharge or action of points?

Leakage of charges near the sharp points of a conductor.

Three Mark Questions

1. Discuss the basic properties of electric charges.

- It is the fundamental property of particles
- SI unit : coulomb.
- The total electric charge in the universe is constant.
- Charge can neither be created nor destroyed.
- The charge q on any object is equal to an integral multiple of this fundamental unit of charge(e).

2. Explain in detail Coulomb's law and its various aspects.

- $\overrightarrow{F_{21}} = k \frac{q_1 q_2}{r^2} \hat{r_{12}}$

  The force on point charge q_2 exerted by another point charge q_1 is $\overrightarrow{F_{21}}$.

  $\hat{r_{21}}$ is directed from q_1 to the charge q_2.

  k is the proportionality constant.

- Difference between Coulomb force and Gravitational force

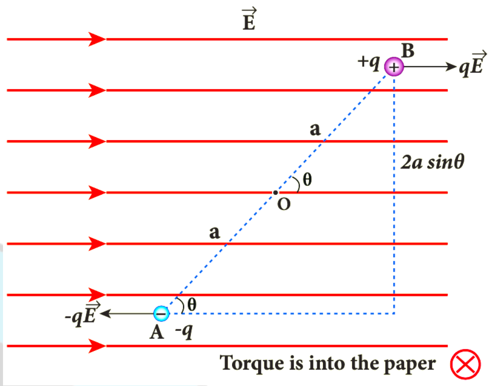
Columb force	Gravitational force
Attractive or repulsive	Always attractive
Dependent on medium	Independent on medium
Comparatively greater force	Comparatively weaker force

3. Define 'electric field' and discuss its various aspects.

- Force experienced by a unit positive charge.
- It is a vector quantity
- Test charge is small.
- There are two kinds of electric field: uniform (constant) electric fields and non-uniform electric field. Uniform electric field will have the same direction and constant magnitude at all points in

space. Non-uniform electric field will have different directions or different magnitudes or both at different points in space.

4. Derive an expression for the torque experienced by a dipole due to a uniform electric field.



The total force acting on a dipole is zero.
These two forces acting at different points will constitute a couple and the dipole experience a torque.

$$\vec{\tau} \ = \ \overrightarrow{OA} \ x(-q\vec{E}) \ + \ \overrightarrow{OB} \ x \ q\vec{E}$$

Magnitude:

$$\tau = | \ \overrightarrow{OA} \ | \ |(-q\vec{E})|sin\theta \ + \ |\overrightarrow{OB}| \ | \ q\vec{E}\ |sin\theta$$

$$\tau \ = \ qE.2asin\theta$$

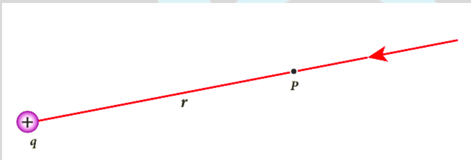
$$\tau \ = \ pEsin\theta$$

Direction:

Using the right cork screw rule torque is perpendicular and directed into the pane of the paper.

$$\underline{\vec{\tau} \ = \ \vec{p}x\vec{E}}$$

5.Derive an expression for electrostatic potential due to a point charge.



The electric potential at a point P is $V = -\int_{\infty}^{r} \vec{E}\,.\,\overrightarrow{dr}$ -> (1)

$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r}$$

$$V = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r}\,.\,\overrightarrow{dr}$$

$$\overrightarrow{dr} \ = \hat{r}dr$$

$$V = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r}\,.\,\hat{r}dr$$

$$\hat{r}\,.\,\hat{r} \ = \ 1$$

$$V = -\frac{1}{4\pi\epsilon_o}\int_{\infty}^{r} \frac{q}{r^2}dr$$

$$-\int_{\infty}^{r} \frac{1}{r^2}dr \ = -\,[-\frac{1}{r}]_{\infty}^{r} \ = \frac{1}{r}$$

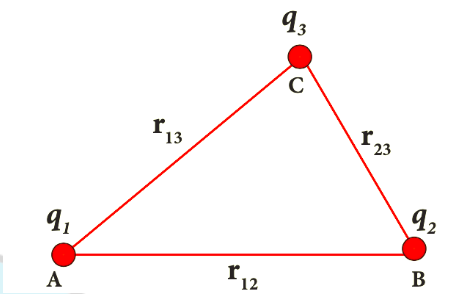
$$V = \frac{1}{4\pi\epsilon_o}\frac{q}{r}$$

6. Obtain an expression for potential energy due to a collection of three point charges which are separated by finite distances.

Work done is stored as potential energy

Work done to bring charge q_1 :

$U_1 = 0$ (No charges in the vicinity)



Work done to bring charge q_2 :

$$U_2 = \frac{1}{4\pi\epsilon_o}\frac{q_1q_2}{r_{12}}$$ (q_1 is present)

Work done to bring charge q_3 :

$$U_3 = \frac{1}{4\pi\epsilon_o}\left(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$$ (q_1 and q_2 present)

Total PE to assemble = $U_1 + U_2 + U_3 = \frac{1}{4\pi\epsilon_o}\left(\frac{q_1q_2}{r_{12}}\ \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right)$

7.Obtain Gauss law from Coulomb's law.

Let us consider a positive point charge Q is surrounded by an imaginary sphere of radius r

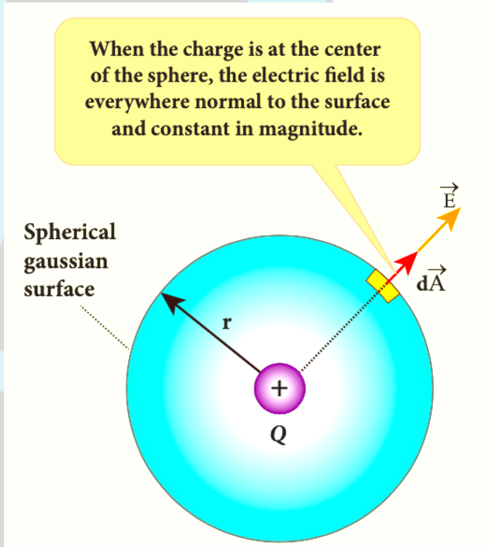
$$\phi_E = \oint \vec{E}.d\vec{A}$$

$$\phi_E = \oint EdAcos\theta$$

$$\left(\theta = o^0\ ,\ cos\theta = 1\right)$$

$$\phi_E = \oint EdA$$

$$\phi_E = E\oint dA$$



Substituting $\oint dA = 4\pi r^2$ and $E = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}$

$$\phi_E = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2} \text{ x } 4\pi r^2$$

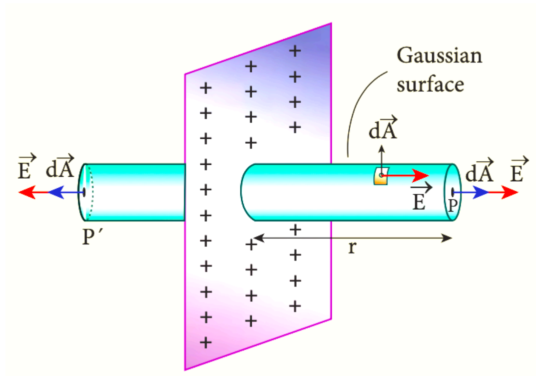
$$= \frac{Q}{\epsilon_o}$$

8.Obtain the expression for electric field due to an charged infinite plane sheet.

Consider an infinite plane sheet of charges with uniform surface charge density σ (charge present per unit area).

Let P be a point at a distance of r from the sheet

Gaussian surface: Cylinder (length = 2r)



From the electric flux definition:

$$\phi_E = \int_{curved\ surface} \vec{E}\,.\,\vec{dA} + \int_P \vec{E}\,.\,\vec{dA} + \int_{P'} \vec{E}\,.\,\vec{dA}$$

$$\int_{curved\ surface} \vec{E}\,.\,\vec{dA} = 0\,(\vec{E} \perp r\ \vec{dA})$$

$$\phi_E = \int_P \vec{E}\,.\,\vec{dA} + \int_{P'} \vec{E}\,.\,\vec{dA}$$

$$\phi_E = 2\int_P EdAcos\theta$$

$$\int_P \vec{E}\,.\,\vec{dA} = \int_{P'} \vec{E}\,.\,\vec{dA} = \int_P EdA \quad \left(\vec{E} \parallel le\ to\ \vec{dA}\right)$$

$$\phi_E = 2E\int_P dA$$

$\phi_E = 2EA$ ——**>(1)**

From the Gauss law:

$$\phi_E = \frac{Q_{enclosed}}{\epsilon_o}$$

$$Q_{enclosed} = \sigma A$$

$\phi_E = \dfrac{\sigma A}{\epsilon_o}$ ——**-(2)**

Combining (1) and (2)

$$2EA = \frac{\sigma A}{\epsilon_o}$$

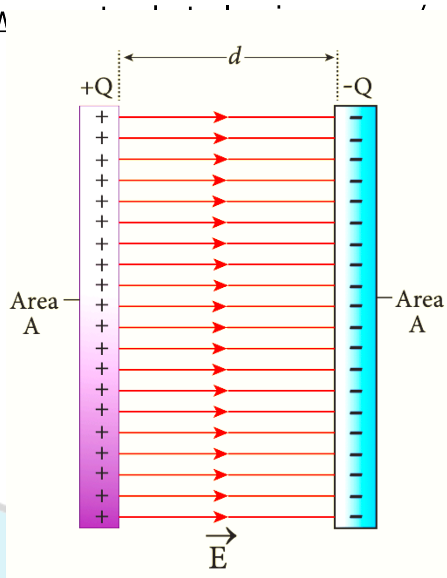
E = $= \dfrac{\sigma}{2\epsilon_o}\hat{n}$ **(in vector form)**

9.Discuss the various properties of conductors in electrostatic equilibrium

- The electric field is zero everywhere inside the conductor.
- There is no net charge inside the conductors.
- The electrostatic potential has the same value on the surface and inside of the conductor.
- The electric field outside the conductor is perpendicular to the surface of the conductor and has a magnitude of $\dfrac{\sigma}{\epsilon_o}$, where σ is the charge density at that point.

10.Explain the process of electrostatic induction.

- Once the charged rod is brought near the conductor, positive charges are located closer to the rod. But the total charge is zero.
- grounding removes the electron from the conducting sphere.
- When the grounding wire is removed from the conductor, the positive charges remain near the charged rod
- By this process, the neutral conducting sphere becomes positively charged.
- This type of charging without actual contact is called electrostatic induction.



11. Obtain the expression for capacitance for a parallel plate capacitor.

Consider a capacitor with two parallel plates each of cross-sectional area *A* and separated by a distance *d*

$$C = \frac{Q}{V}$$

$$C = \frac{Q}{Ed}$$

$$\text{From Gauss law } \left(E = \frac{\sigma}{\epsilon_o}\right)$$

$$C = \frac{\epsilon_o Q}{\sigma d}$$

$$\left(\sigma = \frac{Q}{A}\right)$$

$$C = \frac{A\epsilon_o Q}{Qd} = \frac{A\epsilon_o}{d}$$

12. Obtain the expression for energy stored in the parallel plate capacitor.

To transfer the charge, work is done by the battery

To transfer an infinitesimal charge dQ for a potential difference V, the work done is given by

dW = V dQ

$$V = \frac{Q}{C}$$

$$dW = \frac{Q}{C}\, dQ$$

Upon integrating,

$$\int dW = \int \frac{Q}{C}\, dQ$$

$$W = \frac{1}{C}\left(\frac{Q^2}{2}\right)$$

$$Q = CV$$

$$W = \frac{1}{C}\left(\frac{C^2V^2}{2}\right)$$

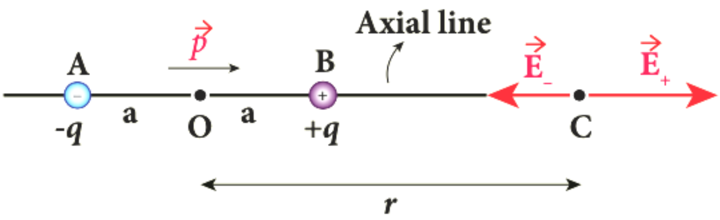
$$W = \frac{1}{2}CV^2$$

Work done is stored as potential energy

$$U_E = \frac{1}{2}CV^2$$

Five *Mark Questions*

1. Calculate the electric field due to a dipole on its axial line



Since +q is located near to C, E^+ is stronger than E^-

The electric field at a point C due to $+q$ is

$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}\hat{p}$$

The electric field at a point C due to $-q$ is

$$\vec{E}_- = -\frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2}\hat{p}$$

Using the superposition principle,

$$\vec{E}_{tot} = \vec{E}_+ + \vec{E}_-$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}\hat{p} - \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2}\hat{p}$$

$$= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right)\hat{p}$$

$$= \frac{1}{4\pi\epsilon_0}q\left(\frac{4ra}{(r^2-a^2)^2}\right)\hat{p}$$

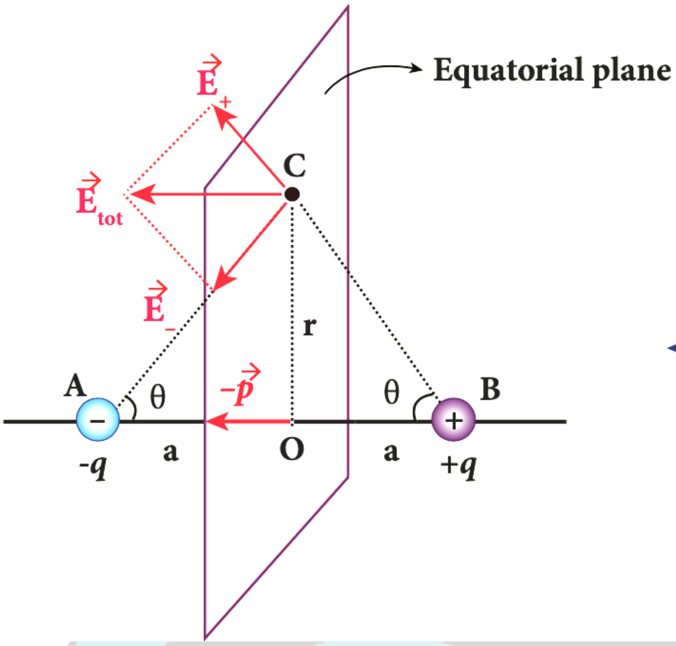
Since 'a' is very small neglect a^2

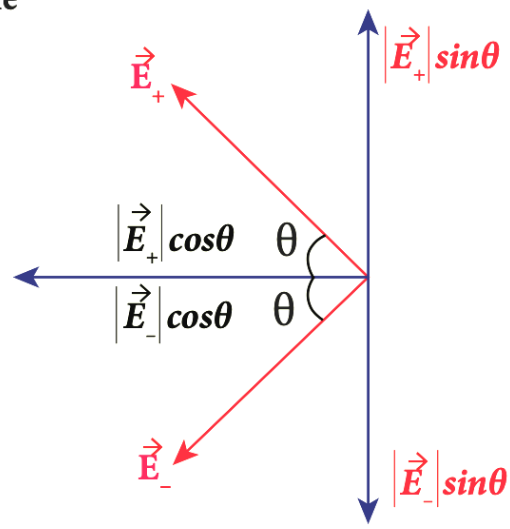
$$= \frac{1}{4\pi\epsilon_0}\left(\frac{4aq}{r^3}\right)\hat{p}$$

($2aq\,\hat{p} = \vec{p}$)

$$\vec{E}_{tot} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3}$$

2. Calculate the electric field due to a dipole on its equatorial plane.





$$\vec{E}_{tot} = -|\vec{E}_+|\cos\theta\,\hat{p} - |\vec{E}_-|\cos\theta\,\hat{p}$$

$$\vec{E}_{tot} = -\frac{1}{4\pi\epsilon_0}\frac{2q\cos\theta}{(r^2+a^2)}\hat{p}$$

since $\cos\theta = \frac{a}{\sqrt{r^2+a^2}}$

$$= -\frac{1}{4\pi\epsilon_0}\frac{2qa}{(r^2+a^2)^{\frac{3}{2}}}\hat{p}$$

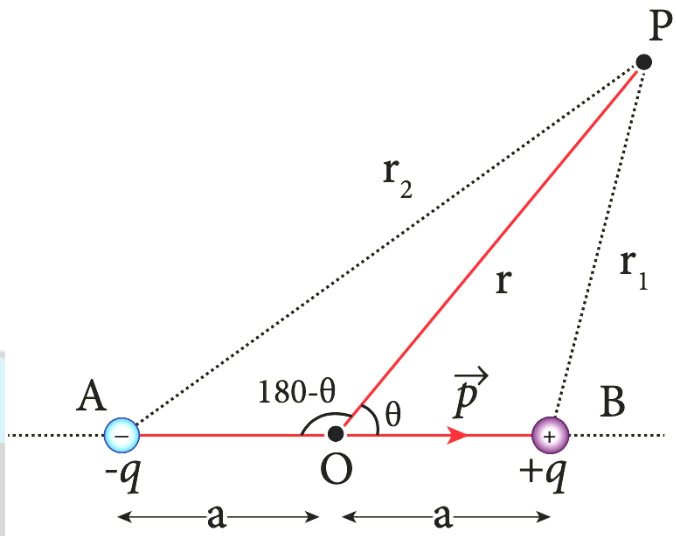
since $\vec{p} = 2qa\hat{p}$

$$\vec{E}_{tot} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{(r^2+a^2)^{\frac{3}{2}}}$$

Since ‘a’ is very small neglect a^2

$$\vec{E}_{tot} = -\frac{1}{4\pi\epsilon_\circ}\frac{\vec{p}}{r^3}$$

3. Derive an expression for electrostatic potential due to an electric dipole.



Total potential at the point P $q = \frac{1}{4\pi\epsilon_\circ}\frac{q}{r_1}$

Potential at P due to charge $-q = -\frac{1}{4\pi\epsilon_\circ}\frac{q}{r_2}$

$$V = \frac{1}{4\pi\epsilon_\circ} q\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

To find $\frac{1}{r_1} = ?$

Using law of cosines ,

$$r_1^2 = r^2\left(1 + \frac{a^2}{r^2} - \frac{2a}{r}\cos\theta\right)$$

Since a is very small, $\frac{a^2}{r^2}$ can be neglected,

$$r_1^2 = r^2\left(1 - 2a\frac{\cos\theta}{r}\right)$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}$$

Using Binomial theorem,

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right)$$

To find $\frac{1}{r_2} = ?$

Replacing θ with $180^o - \theta$ we get r_2,

$$\left(cos(180^o - \theta) \;=\; -\,cos\theta\right)$$

$$\frac{1}{r_2} = \frac{1}{r}\left(1 - a\frac{\cos\theta}{r}\right)$$

$$V = \frac{q}{4\pi\epsilon_\circ}\left(\frac{1}{r}\left(1 + a\frac{\cos\theta}{r} - 1 + a\frac{\cos\theta}{r}\right)\right)$$

$$V = \frac{1}{4\pi\epsilon_\circ}\frac{2aq}{r^2}\cos\theta$$

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

$$V = \frac{1}{4\pi\epsilon_\circ}\frac{\vec{p}\cdot\hat{r}}{r^2} \quad (r >> a)$$

4. **Obtain the expression for electric field due to an infinitely long charged wire.**

Let P be a point located at a perpendicular distance *r* from the wire . The electric field at point P can be found using Gauss law.

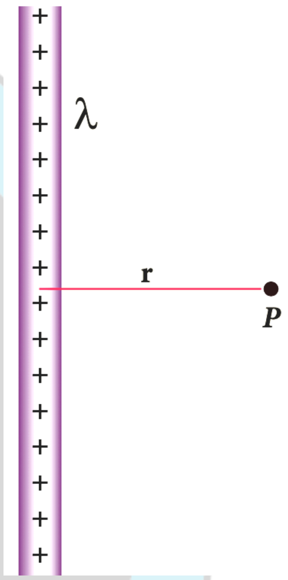
Charge configuration: Infinitely long charged wire.

Charge density: λ (charge per unit length)

Symmetry: Cylindrical

Gaussian Surface: cylindrical Gaussian surface of radius r and length L

From the electric flux definition:



$$\Phi_E = \oint \vec{E}\cdot d\vec{A}$$

$$= \int_{\substack{Curved\\surface}} \vec{E}\cdot d\vec{A} + \int_{\substack{top\\surface}} \vec{E}\cdot d\vec{A} + \int_{\substack{bottom\\surface}} \vec{E}\cdot d\vec{A}$$

- $\int_{top} \vec{E}\,.\,\vec{dA} = \int_{bottom} \vec{E}\,.\,\vec{dA} = 0 \quad \left(\vec{E} \perp r\ \vec{dA}\right)$
- $\int_{curved} \vec{E}\,.\,\vec{dA} = E\int dA \quad \left(\vec{E} \parallel le\ \vec{dA}\right)$

$$\phi_E = E\int dA$$

$= EA$

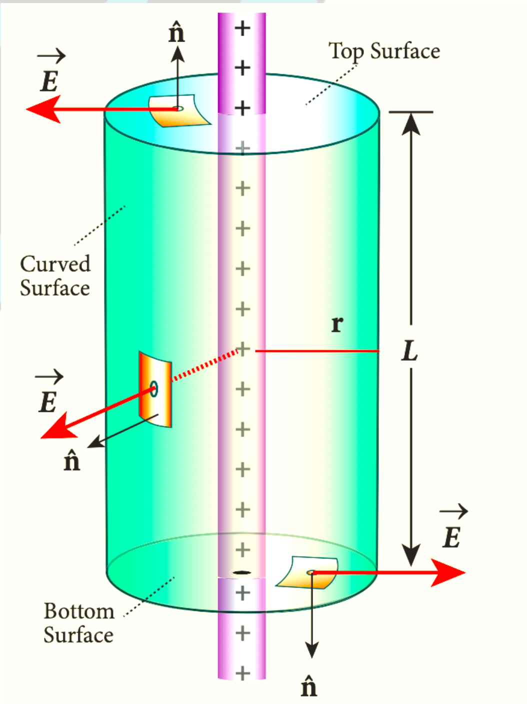
$\phi_E = E\,(2\pi rL)$ ——>(1)

From the Gauss law:

$$\phi_E = \frac{Q_{enclosed}}{\epsilon_o}$$

$= \dfrac{\lambda L}{\epsilon_o}$ ————>(2)

From (1) and (2) we get



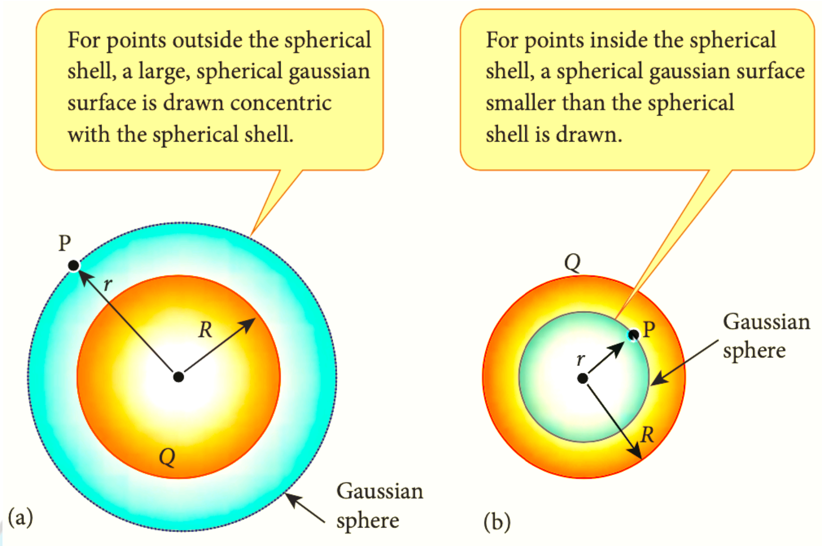
$$E\,(2\pi rL) = \frac{\lambda L}{\epsilon_o}$$

$$E = \frac{\lambda}{2\pi r\epsilon_o}$$

In vector form,

$$\vec{E} = \frac{\lambda}{2\pi r\epsilon_o}\hat{r}$$

5. Obtain the expression for electric field due to a uniformly charged spherical shell.



AT A POINT OUTSIDE THE SHELL:

Charge configuration: Uniformly charged spherical shell with radius R.
Total Charge: Q
Symmetry: Spherical
Gaussian Surface: Gaussian surface of radius *r* is chosen (r > R)
From the electric flux definition:

$$\phi_E = \oint \vec{E} \,.\, d\vec{A} \qquad \left(\vec{E} \parallel le \ d\vec{A} \right)$$

$$\phi_E = E \oint dA \quad = \text{EA} \qquad \phi_E = \text{E}\,(4\pi r^2) \text{———>(1)}$$

From Gauss law:

$$\phi_E = \frac{Q_{enclosed}}{\epsilon_o} \qquad \phi_E = \frac{Q}{\epsilon_o} \text{------>(2)}$$

Combining (1) and (2)

$$\text{E}(4\pi r^2) = \frac{Q}{\epsilon_o}$$

$$\text{E} = \frac{Q}{4\pi r^2 \epsilon_o}$$

In vector form, $\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}\hat{r}$

AT A POINT ON THE SHELL:

Take r = R

We get, $\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{Q}{R^2}\hat{r}$

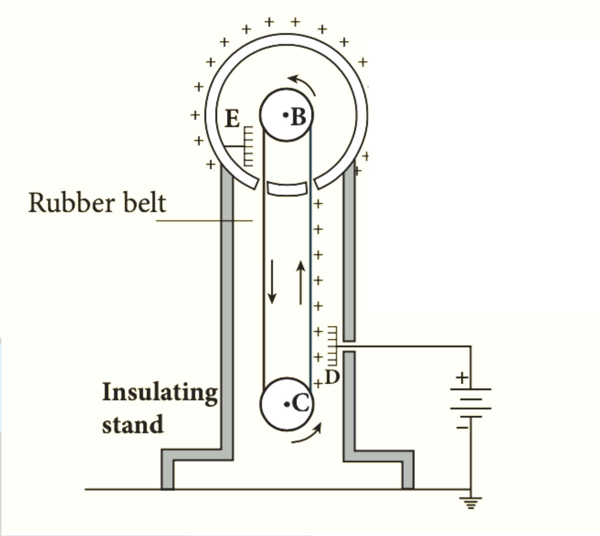
AT A POINT INSIDE THE SHELL:

$Q_{enclosed} = 0$
E = 0

6.Explain in detail the construction and working of a Van de Graaff generator.

Principle:

Electrostatic induction and action of points



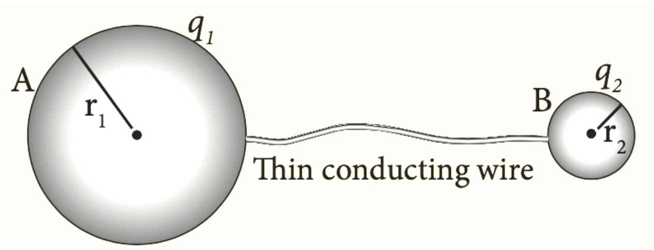
Construction:

- The spherical conductor is fixed on the insulating stand
- Pulley B:
  - Mounted at the centre of the hollow sphere
  - metallic conductors E near the pulley
- Pulley C:
  - Fixed at the bottom
  - Metallic conductors D near the pulley
  - Driven continuously by the electric motor.
- Belt:
  - Insulating materials like silk or rubber
- Comb D
  - Potential of 10^4 V by a power supply.
- Comb E
  - connected inner side of the hollow metal sphere.

Working:

- Comb D gets ionized.
- The positive charges stick to the belt and move up.
- Sphere acquires a positive charge due to electrostatic induction
- The positive charges are distributed uniformly on the outer surface of the hollow sphere.
- At the same time, the negative charges nullify the positive charges in the belt due to corona discharge before it passes over the pulley.
- When the belt descends, it has almost no net charge
- This process continues until the outer surface produces the potential difference of the order of 10^7 which is the limiting value.
- Leakage of charges is reduced by enclosing the machine in a gas-filled steel chamber at very high pressure.
- Used to accelerate positive ions.

7. Explain in detail how charges are distributed in a conductor, and the principle behind the lightning conductor.



The distance between the spheres is much greater than the radii of either spheres

The electrostatic potential at the surface of the sphere A is given by

$$V_A = \frac{1}{4\pi\epsilon_\circ}\frac{q_1}{r_1}$$

The electrostatic potential at the surface of the sphere B is given by

$$V_B = \frac{1}{4\pi\epsilon_\circ}\frac{q_2}{r_2}$$

Since the spheres are connected by the conducting wire, the surfaces of both spheres together form an equipotential surface.

$$V_A = V_B$$

$$\frac{q_1}{r_1} = \frac{q_2}{r_2}$$

Let the charge density on the surface of sphere A be σ_1 and that on surface B be σ_2
This implies that $q_1 = 4\pi r_1^2 \sigma_1$ and $q_2 = 4\pi r_2^2 \sigma_2$

$$\sigma_1 r_1 = \sigma_2 r_2$$

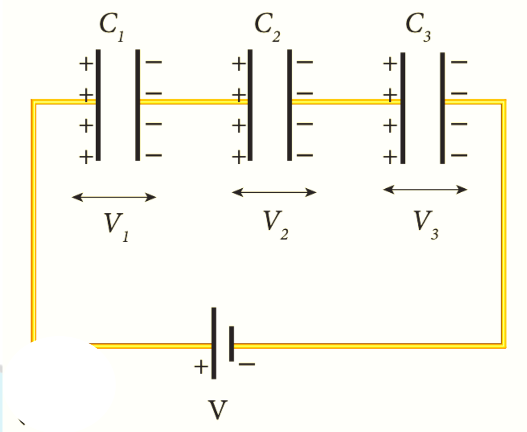
$$\sigma r = constant$$

Lightning arrestor:

- It works on the principle of action at points.
- This device consists of a long thick copper rod passing from top of the building to the ground.
- The lower end of the rod is connected to copper plate which is buried deep into the ground.
- When a negatively charged cloud is passing above the building, it induces a positive charge on the spike.
- Since the induced charge density on thin sharp spike is large, it results in a corona discharge.
- This positive charge ionizes the surrounding air which in turn neutralizes the negative charge in the cloud.
- The negative charge pushed to the spikes passes through the copper rod and is safely diverted to the Earth.
- The lightning arrester does not stop the lightning; rather it diverts the lightning to the ground safely.

8.Derive the expression for resultant capacitance, when capacitors are connected in series and in parallel.

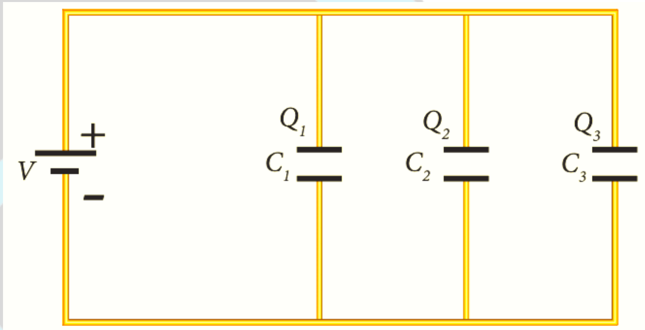
Capacitors in series:



$$V = V_1 + V_2 + V_3$$
$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$
$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors in parallel:

$$Q = Q_1 + Q_2 + Q_3$$
$$C_pV = C_1V + C_2V + C_3V$$
$$C_p = C_1 + C_2 + C_3$$



9.Explain in detail the effect of a dielectric placed in a parallel plate capacitor.

S. No	Dielectric is inserted	Charge Q	Voltage V	Electric field E	Capacitance C	Energy U
1	When the battery is disconnected	Constant	decreases	Decreases	Increases	Decreases
2	When the battery is connected	Increases	Constant	Constant	Increases	Increases

10.Explain dielectrics in detail and how an electric field is induced inside a dielectric.

A dielectric is a non-conducting material and has no free electrons.

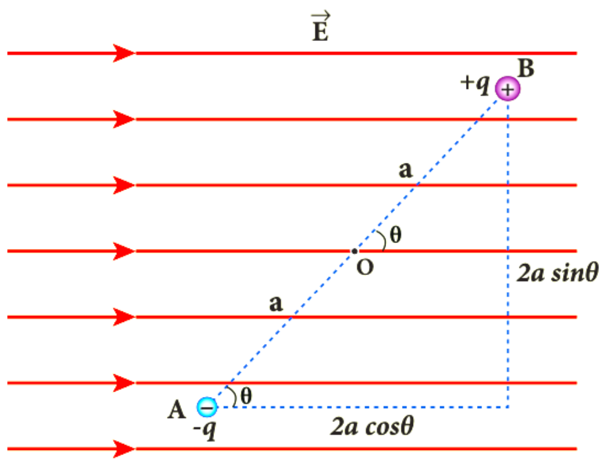
When an external electric field is applied, the electrons are not free to move anywhere but they are realigned in a specific way

The external electric field only realigns the charges so that an internal electric field is produced.

The magnitude of the internal electric field is smaller than that of the external electric field

Therefore the net electric field inside the dielectric is not zero but is parallel to an external electric field with a magnitude less than that of the external electric field.

11.Derive an expression for electrostatic potential energy of the dipole in a uniform electric field.



Work done in rotating the dipole through $d\theta$,

$dw = \tau . d\theta$

$= pE \sin\theta . d\theta$

The total work done in rotating the dipole through an angle θ is

$W = \int dw$

$W = pE \int \sin\theta . d\theta = -pE \cos\theta$

This work done is the potential energy (U) of the dipole.

$\therefore \quad U = -pE \cos\theta$

All the Best