

Complex numbers**Choose the correct answer:-****5 x 1 = 5**

1) If z is a complex number of unit modulus and argument θ , then $\arg \frac{1+z}{1+\bar{z}}$ is equal to

- (1) $-\theta$ (2) $90^\circ - \theta$ (3) θ (4) $180^\circ - \theta$

2) If $\arg(z) < 0$, then $\arg(-z) - \arg(z)$ equals to

- (1) π (2) $-\pi$ (3) $-\frac{\pi}{2}$ (4) $\frac{\pi}{2}$

3) If w is an imaginary solution of cube roots of unity and $(1+w)^7 = A + Bw$

Then A and B respectively are

- (1) $0, 1$ (2) $1, 1$ (3) $1, 0$ (4) $-1, -1$

4) The value of $i + i^{23} + i^{24} + i^{25} + i^{26}$ is

- (1) i (2) -1 (3) 0 (4) $-i$

5) If $(m-5) + i(n+4)$ is the complex conjugate of $(2m+3) + i(3n-2)$ then (n, m) are

- (1) $(-8, \frac{-1}{2})$ (2) $(\frac{-1}{2}, 8)$ (3) $(\frac{1}{2}, -8)$ (4) $(\frac{-1}{2}, -8)$

Answer any five of the following (Question no 13 is compulsory):-**5 x 2 = 10**

6) Show that $|3z - 5 + i| = 4$ represents a circle, find its radius and centre.

7) Obtain the cartesian form of the locus $z = x + iy$ in the following case: $|z - 1| = |z + i|$.

8) State true or false:-

a) Any complex number $z = x + iy$ can be expressed as $z = r(\cos\theta + i\sin\theta)$.

b) The n^{th} roots of unity are in arithmetic progression.

9) Find the polar form of the complex number $-2 - i2$.

10) Simplify $i^2 i^3 \dots i^{2000}$.

11) If $z = 2 + 5i$ find a) Additive inverse of z .

b) multiplicative inverse of z .

12) Find the value of $\sum_{k=1}^8 \cos \frac{2k\pi}{9} + i \sin \frac{2k\pi}{9}$.

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13) If $z^2 = (0,1)$ find z . [Hint:- Find the square root of z]

Answer any five of the following (Question no 19 is compulsory):-

5 x 3 = 15

14) If $w \neq 1$ the cube roots of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are $-1, 1 - 2w, 1 - 2w^2$.

15) Show that $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

16) Prove that $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$.

17) If $\cos A + \cos B + \cos C = \sin A + \sin B + \sin C = 0$, show that $\cos 3A + \cos 3B + \cos 3C = 3\cos(A+B+C)$.

18) If $\frac{1+z}{1-z} = \cos 2A + i \sin 2A$, show that $z = i \tan A$.

19) Find the value of

$$\left(\frac{1 + \sin \frac{\pi}{10} + i \cos \frac{\pi}{10}}{1 + \sin \frac{\pi}{10} - i \cos \frac{\pi}{10}} \right)^{10}$$

20) Solve $x^4 - 1 = 0$.

Answer the following:-

3 x 5 = 15

21) State and prove the triangle inequality

OR

Find all the cube roots of $\sqrt{3} + i$.

22) Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$.

OR

State and prove the triangle inequality

23) If $z = x + iy$ and $\arg \frac{z-1}{z+1} = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$.

OR

Given the complex $z = 3 + 2i$, represents the complex number z, iz and $z + iz$ in an argand plane. Show that these complex numbers form the vertices of an isosceles triangle.

ALL THE BEST