UNIT-2 QUANTUM MECHANICAL MODEL OF ATOM

MY REVISION TIMELINE:-

SUMMARY:-

- Bohr atom model: (For hydrogen atom)
  - Energy of an electron is quantised.
  - Electron revolve around nucleus in fixed circular stationary orbit.
  - Electrons can revolve only in orbitals in which mvr = $\frac{nh}{2\pi}$ (Angular momentum of e^-) where n=1, 2, 3,….
  - When an e^- jumps from higher energy (E_2) to lower energy (E_1), the excess energy is emitted as radiation. Conversely when energy is supplied it will jump from E_1 to E_2.
  - $E_2 - E_1 = h\nu \Rightarrow \nu = \frac{E_2 - E_1}{h}$
  - Note: As long as e^- remains in the same orbit it does not lose or gain energy.
- The results of Bohr atom model:
  - $E_n = \frac{-13.6z^2}{n^2}$ eV/atom
  - $r_n = \frac{(0.529)n^2}{z}$ Å
  - $E_n = \frac{-(1312.8)z^2}{n^2}$ kJ/mol

    Where r_n is the radius of n^{th} orbit and E_n is the energy of electron in n^{th} orbit
  - Note: Formula is applicable only to hydrogen and hydrogen like species i.e., He^+ and Li^{2+} etc..
- The electron in the orbit is characterized by a set of four quantum numbers
  - Principal quantum number (n)
  - Azimuthal quantum number (l)
  - Magnetic quantum number (m_l)
  - Spin quantum number (m_s)
- Shapes of orbitals:
  - For s-orbital l=0 and corresponding m value is 0
  - For p-orbital l=1 and corresponding m values are -1, 0, +1 (i.e.) p_x, p_y, p_z
  - For d-orbital l=2 and corresponding m values are -2, -1, 0, +1, +2 (i.e.) $d_{x^2-y^2}$, d_{z^2}, d_{xy}, d_{yz}, d_{zx}
  - For 'F' orbital l=3 and corresponding m values are -3, -2, -1, 0, +1, +2, +3 (i.e.) F_{z^3}, F_{xz^2}, F_{yz^2}, F_{xyz}, $F_{z(x^2-y^2)}$, $F_{x(x^2-3y^2)}$, $F_{y(3x^2-y^2)}$
- As n value increases energy of orbital increases.

- Based on the (n+l) rule, the increasing order of energy of orbitals is $1s<2s<2p<3s<3p<4s<3d<4p<5s<4d<5p<6s<4F<5d<6p<7s<5F<6d$
- The order of effective nuclear charge felt by an electron in an orbital within the given shell is s>p>d>F
- Within a given energy level, the energy of the orbitals are in the order $s<p<d<F$
- In an atom electrons are filled in various orbitals according to Aufbau principle, Pauli exclusion principle and Hund's rule
- Exactly half filled and completely filled orbitals have greater stability than other partially filled configurations in degenerate orbitals.

FORMULAS:-

- $v = \frac{c}{\lambda}$
- $E = \frac{hc}{\lambda}$
- $E_n = \frac{-13.6z^2}{n^2}$ eV/atom
- $r_n = \frac{(0.529)n^2}{z}$ Å
- $E_n = \frac{-(1312.8)z^2}{n^2}$ kJ/mol
- E = hv (Planck's equation)
- $E = mc^2$ (Einstein's equation)
- $\lambda = \frac{h}{mv}$ (de-Broglie equation)
- Heisenberg's uncertainty principle: $\Delta x.\Delta p \geq \frac{h}{4\pi}$ (or) $\Delta x.(m\Delta v) \geq \frac{h}{4\pi}$
- No. of electrons in a shell: $2n^2$
- No. of electrons in a sub-shell: 2(2l+1)
- Angular momentum: $\sqrt{l(l+1)}\frac{h}{2\pi}$
- Number of radial nodes: n-l-1
- Number of angular nodes: l

HINTS TO SOLVE PROBLEMS:-

- $h = 6.626 \times 10^{-34}$ Js
-

Name of the constant	Unit	Electron (e^-)	Proton (p^+)	Neutron (n)
Mass	Kg	9.019×10^{-31}	1.673×10^{-27}	1.675×10^{-27}
Charge (e)	Coulomb	-1.602×10^{-19}	1.602×10^{-19}	Zero
Specific charge (e/m)	C/Kg	1.76×10^{11}	9.58×10^{7}	Zero
Specific charge Order: $n < a < p^+ < e^-$				

TEXTBOOK EVALUATION

Multiple choice questions:-

1. Electronic configuration of species M^{2+} is $1s^2\ 2s^2\ 2p^6 3s^2\ 3p^6\ 3d^6$ and its atomic weight is 56. The number of neutrons in the nucleus of species M is

(a) 26 (b) 22
(c) 30 (d) 24

Explanation:
M^{2+} : $1s^2\ 2s^2\ 2p^6 3s^2\ 3p^6\ 3d^6$
M : $1s^2\ 2s^2\ 2p^6 3s^2\ 3p^6\ 3d^8$
Atomic number = 26
Mass number = 56
No. of neutrons = 56 – 26

No. of neutrons = 30

2. The energy of light of wavelength 45 nm is

(a) 6.67 x 10^{15} J (b) 6.67 x 10^{11} J
(c) 4.42 .x 10^{18} J (d) 4.42 x 10^{-15} J

Explanation:
E = hv
$E = \frac{hc}{\lambda}$
$E = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{45 \times 10^{-9}}$

$E = 4.42 \times 10^{-18}$ J

3. The energies E_1 and E_2 of two radiations are 25 eV and 50 eV respectively. The relation between their wavelengths i.e. λ_1 and λ_2 will be

(a) $\frac{\lambda 1}{\lambda 2} = 1$ **(b) $\lambda_1 = 2\ \lambda_2$**
(c) $\lambda_1 = \sqrt{25 \times 50}\ \lambda 2$ (d) $2\ \lambda_1 = \lambda_2$

Explanation:
$$\frac{E_1}{E_2} = \frac{25eV}{50eV} = \frac{1}{2}$$
$$\frac{hc}{\lambda_1} \times \frac{\lambda_2}{hc} = \frac{1}{2}$$

$\lambda_1 = 2\ \lambda_2$

4. Splitting of spectral lines in an electric field is called

(a) Zeeman effect (b) Shielding effect
(c) Compton effect **(d) Stark effect**

Explanation:
Splitting of spectral lines in magnetic field is called “Zeeman effect” and Splitting of spectral lines in electric field is called “Stark effect”

5. Based on equation E = -2.178 x 10^{18} J (z2n2) certain conclusions are written. Which of them is not correct ? (NEET)

(a) Equation can be used to calculate the change in energy when the electron changes orbit

(b) For n = 1, the electron has a more negative energy than it does for n = 6 which means that the electron is more loosely bound in the smallest allowed orbit

(c) The negative sign in equation simply means that the energy of electron bound to the nucleus is lower than it would be if the electrons were at the infinite distance from the nucleus.

(d) Larger the value of n, the larger is the orbit radius.

Explanation:

For n=1, the electron has more negative energy than it does for n=6. Which means that the electron is strongly bound in the smallest allowed orbit.

6. According to the Bohr Theory, which of the following transitions in the hydrogen atom will give rise to the least energetic photon?

(a) n = 6 to n = 1 (b) n = 5 to n = 4

(c) n = 5 to n = 3 **(d) n = 6 to n = 5**

Explanation:

n = 6 to n = 5

$E_6 = -13.6 / 6^2$; $E_5 = -13.6 / 5^2$

$E_6 - E_5 = (-13.6 / 6^2) - (-13.6 / 5^2) = 0.166$ eV atom^{-1}

$E_5 - E_4 = (-13.6 / 5^2) - (-13.6 / 4^2) = 0.306$ eV atom^{-1}

7. Assertion : The spectrum of He^+ is expected to be similar to that of hydrogen

Reason : He^+ is also one electron system,

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false

(d) If both assertion and reason are false

8. Which of the following pairs of d-orbitals will have electron density along the axes ? (NEET Phase – II)

(a) d_z^2, d_{xz} (b) d_{xz}, d_{yz}

(c) d_z^2, d_{x2-y2} (d) d_{xy}, d_{x2-y2}

9. Two electrons occupying the same orbital are distinguished by

(a) azimuthal quantum number **(b) spin quantum number**

(c) magnetic quantum number (d) orbital quantum number

10. The electronic configuration of Eu (atomic no. 63) Gd (atomic no. 64) and Tb (atomic no. 65) are (NEET – Phase II)

(a) [Xe] $4f^7 5d^1 6s^2$, [Xe] $4f^7 5d^1 6s^2$ and [Xe] $4f^8 5d^1 6s^2$

(b) [Xe] $4f^7$, $6s^2$, [Xe] $4f^7 5d^1 6s^2$ and [Xe] $4f^9 6s^2$

(c) [Xe] $4f^7$, $6s^2$, [Xe] $4f^8 6s^2$ and [Xe] $4f^8 5d^1 6s^2$

(d) [Xe] $4f^8 5d^1 6s^2$[Xe] $4f^7 5d^1 6s^2$ and [Xe] $4f^9 6s^2$

11. The maximum number of electrons in a sub shell is given by the expression

(a) $2n^2$ (b) 21 + 1

(c) 41 + 2 (d) none of these

12. For d-electron, the orbital angular momentum is

(a) $\frac{\sqrt{2h}}{2\pi}$ (b) $\frac{\sqrt{2h}}{2\pi}$

(c) $\frac{\sqrt{2\times4}}{2\pi}h$ **(d)** $\boldsymbol{\frac{\sqrt{6}}{2\pi}h}$

Explanation:

Orbital angular momentum = $\sqrt{l(l+1)}\,\frac{h}{2\pi}$

For d orbital, Orbital angular momentum = $\sqrt{2(2+1)}\,\frac{h}{2\pi}$

For d orbital, Orbital angular momentum = $\boldsymbol{\sqrt{6}\,\frac{h}{2\pi}}$

13. What is the maximum numbers of electrons that can be associated with the following set of quantum numbers? n = 3,l = 1 and m = -1

(a) 4 (b) 6

(c) 2 (d) 10

Explanation:

n = 3; l = 1; m = -1 either $3p_x$ or $3p_y$

14. Assertion: Number of radial and angular nodes for 3p orbital are l, l respectively. Reason: Number of radial and angular nodes depends only on principal quantum number.

(a) both assertion and reason are true and reason is the correct explanation of assertion.

(b) both assertion and reason are true but reason is not the correct explanation of assertion.

(c) assertion is true but reason is false

(d) both assertion and reason are false

Explanation:

No. of radial node = n- l – 1

No. of angular node = l for 3p orbital

No. of angular node = l =1

No. of radial node = n- l – 1 = 3 – 1 – 1 = 1

15. The total number of orbitals associated with the principal quantum number n = 3 is

(a) 9 (b) 8

(c) 5 (d) 7

Solution:

n = 3; l = 0; m_1 = 0: one s orbital

n = 3; l = 1; m_1 = -1, 0, 1: three p orbitals

n = 3; l = 2; m_1 = -2, -1, 0, 1, 2: five d orbitals

Total number of orbitals = 9

16. If n = 6, the correct sequence for filling of electrons will be

(a) ns → (n – 2) f → (n – 1)d → np (b) ns → (n – 1) d → (n – 2) f → np

(c) ns → (n – 2) f → np → (n – 1) d (d) none of these are correct

Explanation:

n = 6 According Aufbau principle,

6s → 4f → 5d → 6p

ns → (n – 1)f → (n – 2)d → np.

17. Consider the following sets of quantum numbers:

	n	l	m	s
i)	3	0	0	$+\frac{1}{2}$
ii)	2	2	1	$-\frac{1}{2}$
iii)	4	3	-2	$+\frac{1}{2}$
iv)	1	0	-1	$+\frac{1}{2}$
v)	3	4	3	$-\frac{1}{2}$

Which of the following sets of quantum number is not possible?

(a) (i), (ii), (iii) and (iv)
(b) (ii), (iv) and (v)
(c) (z) and (iii)
(d) (ii), (iii) and (iv)

Explanation:

(ii) l can have the values from 0 to n – 1 n = 2; possible l values are 0, 1 hence l = 2 is not possible.

(iv) for l = 0; m = -1 not possible

(v) for n = 3 l = 4 and m = 3 not possible.

18. How many electrons in an atom with atomic number 105 can have (n + 1) = 8?

(a) 30
(b) 17
(c) 15
(d) unpredictable

Explanation:

Electronic configuration of an atom with atomic number 105 is [Rn] $5f^{14}$ $6d^3$ $7s^2$

Orbital	**(n+1)**	**No. of electrons**
5f	5+3=8	14
6d	6+2=8	3
7s	7+0=7	2

Number of electrons = 17

19. Electron density in the yz plane of 3 $d_{x^2-y^2}$ orbital is

(a) zero
(b) 0.50
(c) 0.75
(d) 0.90

20. If uncertainty in position and momentum are equal, then minimum uncertainty in velocity is

(a) $\frac{1}{m}\sqrt{\frac{h}{\pi}}$
(b) $\sqrt{\frac{h}{\pi}}$
(c) $\frac{1}{2m}\sqrt{\frac{h}{\pi}}$
(d) $\frac{h}{4\pi}$

Explanation:

$\Delta x.\Delta p \geq \frac{h}{4\pi}$

$\Delta x.\Delta p \geq \frac{h}{4\pi}$

$\Delta p^2 \geq \frac{h}{4\pi}$

$m^2(\Delta v)^2 \geq \frac{h}{4\pi}$

$(\Delta v) \geq \sqrt{\frac{h}{4\pi m^2}}$

$$\boxed{(\Delta v) \geq \frac{1}{2m}\sqrt{\frac{h}{\pi}}}$$

21. A macroscopic particle of mass 100 g and moving at a velocity of 100 cm s^{-1}d will have a de Broglie wavelength of

(a) 6.6 x 10^{-29} cm
(b) 6.6 x 10^{-30} cm
(c) 6.6 x 10^{-31} cm
(d) 6.6 x 10^{-32} cm

Explanation:

m = 100 g = 100 x 10^{-3} kg

v = 100 cm s^{-1} = 100 x 10^{-2} m s^{-1}

$\lambda = \frac{h}{mv}$

$\lambda = \frac{6.626 \times 10^{34}}{100 \times 10^{-3} \times 100 \times 10^{-2}}$

$\lambda = 6.626 \times 10^{-33}$ ms^{-1}

$$\boxed{\lambda = 6.626 \times 10^{-31}\ cms^{-1}}$$

22. The ratio of de Broglie wavelengths of a deuterium atom to that of an a – particle, when the velocity of the former is five times greater than that of later, is

(a) 4
(b) 0.2
(c) 2.5
(d) 0.4

Explanation:

$\frac{\lambda_D}{\lambda_\alpha} = \frac{m_\alpha v_\alpha}{m_D v_D}$

$\frac{\lambda_D}{\lambda_\alpha} = \frac{4 \times v}{2 \times 5 \times v}$

$\frac{\lambda_D}{\lambda_\alpha} = \frac{2}{5}$

$$\boxed{\frac{\lambda_D}{\lambda_\alpha} = 0.4}$$

23. The energy of an electron in the 3rd orbit of hydrogen atom is -E. The energy of an electron in the first orbit will be

(a) – 3E
(b) – E /3
(c) – E / 9
(d) – 9E

Explanation:

$E_n = \frac{-13.6}{n^2} eV$ atom^{-1}

$E_1 = \frac{-13.6}{1^2} = -13.6$

$E_3 = \frac{-13.6}{3^2} = \frac{-13.6}{9}$

Given that $E_3 = -E$

$\frac{-13.6}{9} = -E$

$-13.6 = -9E$

$\boxed{=> E_1 = -9E}$

24. Time independent Schnodinger wave equation is

(a) $\hat{H}\psi = E\psi$

(b) $\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E+V)\,\psi = 0$

(c) $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2m}{h^2}(E-V)\,\psi = 0$

(d) all of these

25. Which of the following does not represent the mathematical expression for the Heisenberg uncertainty principle?

(a) $\Delta E.\Delta p \geq h/4\pi$

(b) $\Delta E.\Delta v \geq h/4\pi m$

(c) $\Delta E.\Delta t \geq h/4\pi$

(d) $\Delta E.\Delta x \geq h/4\pi$

<u>Write brief answers to the following questions:-</u>

26. Which quantum number reveal information about the shape, energy, orientation and size of orbitals?

- **Principal** quantum number defines **energy** and **size** of an orbital.
- **Azimuthal** quantum number defines **shape** of an orbital.
- **Magnetic** quantum number defines **spatial orientation** of an orbital.

27. How many orbitals are possible for n =4?

If n = 4, the possible number of orbitals are calculated as follows:

If n = 4, l values are 0, 1, 2, 3

If l = 0, m = 0 = 1 orbital

If l = 1, m = -1,0, +1 = 3 orbitals

If l = 2, m = -2,-1,0, +1,+2 = 5 orbitals

If l = 3, m = -3,-2,-1,0, +1,+2,+3 = 7 orbitals

∴ Total number of orbitals = 16 orbitals

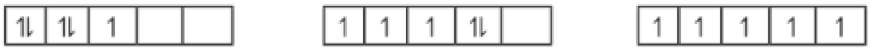
28. How many radial nodes for 2s, 4p, 5d and 4f orbitals exhibit? How many angular nodes?

Orbital	**n**	**l**	**Radial node (n-l-1)**	**Angular node (l)**
2s	2	0	1	0
4p	4	1	2	1
5d	5	2	2	2
4f	4	3	0	3

29. The stabilisation of a half filled d - orbital is more pronounced than that of the p-orbital why?

- **Energy electrons symmetry:**
  The **half filled orbitals** are **more symmetrical** than partially filled orbitals and this symmetry leads to **greater stability**.
- **Exchange energy:**
  The electrons with same spin in the different orbitals of the same sub shell can exchange their position. During exchange process the energy is released and this energy is known as **exchange energy**. Greater the **number of exchanges**, greater the **exchange energy** and hence **greater** the **stability**.
  In **d-orbital**, 10 exchanges are possible but in **p-orbital** 6 exchanges are possible. So, d – orbital with **5 unpaired** electrons (10 exchanges) i.e. half filled is more stable than p – orbital with **3 unpaired** electrons (6 exchanges).

30. Consider the following electronic arrangements for the d^5 configuration.



(a) (b) (c)

(i) which of these represents the ground state
(ii) which configuration has the maximum exchange energy.
(c) represents **ground state**
(a) represents **maximum energy state**

31. State and explain pauli exclusion principle.
- Pauli's exclusion principle states that **"No two electrons in an atom can have the same set of values of all four quantum numbers".**
- **Explanation:**
  Each electron must have **unique values** for the four quantum numbers (n, l, m, s).
- **Illustration:**
  H (Z = 1) $1s^1$.
  One electron is present in hydrogen atom, the four quantum numbers are **n = 1, l = 0, m = 0 and s = + ½.** For helium Z = 2. He: $1s^2$. In this one electron has the quantum number same as that of hydrogen, **n = 1, l = 0, m = 0 and s = +½.** For other electron, fourth quantum number is different, i.e. **n = 1, l = 0, m = 0 and s = – ½.**

32. Define orbital? What are the n and l values for $3p_x$ and $4d_{x^2-y^2}$ electron?
The solution to **Schrodinger** equation gives permitted energy values called **eigen values** and the **wave functions** corresponding to the given eigen values are called atomic orbitals.
For $3p_x$ – n=3, l=1
For $4d_{s^2-y^2}$ – n=4, l=2

33. Explain briefly the time independent schrodinger wave equation?
- Erwin Schrodinger expressed the wave nature of electron in terms of a differential equation.
- The equation determines the change of wave function in space depending on the field of force in which the electron moves.
- The time independent Schrodinger equation can be expressed as
  $\hat{H}\psi = E\psi \rightarrow 1$
  Where $\hat{H}$ is called **Hamiltonian operator,** ψ is **wave function** and **E** is the **energy of the system.**
-

$$\hat{H} = \left[\frac{-h^2}{8\pi^2 m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)+V\right] \quad \rightarrow 2$$

- Equation 1 can be written as

$$\left[\frac{-h^2}{8\pi^2 m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+V\psi\right] = E\psi \quad \rightarrow 3$$

Multiply by $\frac{-8\pi^2 m}{h^2}$ and rearranging

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+\frac{8\pi^2 m}{h^2}(E-V)\psi = 0 \quad \rightarrow 4$$

- The above Schrodinger wave equation does not contain time as a variable and is referred to as time independent **Schrodinger wave equation**.
- The equation can be solved only for certain values of E, the total energy i.e. the **energy of the system is quantised**.
- The permitted total energy values are called **eigen values** and corresponding wave functions represent the **atomic orbitals**.

34. Calculate the uncertainty in position of an electron, if Δv = 0.1% and υ = 2.2 x10^6 ms^{-1}.

Given:

$\Delta v = 0.1\%$

$v = 2.2 \times 10^6$ ms^{-1}

$m = 9.1 \times 10^{-31}$kg

$\Delta x = ?$

Formula used:

$\Delta x.\Delta p \geq \frac{h}{4\pi}$

Solution:

$\Delta x.\Delta p \geq 5.28 \times 10^{-35}$

$\Delta x.(m.\Delta v) \geq 5.28 \times 10^{-35}$

$\Delta v = \frac{0.1}{100} \times 2.2 \times 10^6$

$\Delta v = 2.2 \times 10^3$ m

$$\therefore \Delta x \geq \frac{5.28 \times 10^{-35}}{9.1 \times 10^{-31} \times 2.2 \times 10^3}$$

$$\boxed{\Delta x \geq 2.64 \times 10^{-8}\ m}$$

35. Determine the values of all the four quantum numbers of the 8th electron in O$^-$ atom and 15th electron in Cl atom.

- Electron configuration of O: **$1s^2\ 2s^2\ 2p^4$**
  8th electron is present in $2p_x$ orbital and the quantum numbers are **n = 2, l = 1, m = either +1 or -1 and s = -1/2**
- Electron configuration of Cl: **$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^5$**
  15th electron is present in $3p_z$ orbital and the quantum numbers are **n = 3, l = 1, m = either +1 or -1 and s = +1/2**

36. The quantum mechanical treatment of the hydrogen atom gives the energy value:

$E_n = \frac{-13.6}{n^2}$ eV atom^{-1}

i) use this expression to find ΔE between n = 3 and n=4

ii) Calculate the wavelength corresponding to the above transition.

Given:

$E_n = \frac{-13.6}{n^2}$ eV atom^{-1}

Solution:

- When n = 3, E_3 = −13.632 = −13.69 = – 1.511 eV atom^{-1}
  When n = 4, E_4 = −13.642 = – 0.85 eV atom^{-1}
  $\Delta E = E_4 - E_3 = -0.85 - (-1.511)$

$$\boxed{\Delta E = +\ 0.661\ eV\ atom^{-1}}$$

- $\Delta E = 0.66 \times 1.6 \times 10^{-19}$ J $= 1.06 \times 10^{-19}$ J

$$\frac{hc}{\lambda} = \Delta E$$

$$\frac{hc}{\lambda} = 1.06 \times 10^{-19}$$

$$\lambda = \frac{hc}{1.06 \times 10^{-19}}$$

$$\lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.06 \times 10^{-19}}$$

$$\boxed{\lambda = 1.875 \times 10^{-6}\ m}$$

37. How fast must a 54g tennis ball travel in order to have a de Broglie wavelength that is equal to that of a photon of green light 5400Å?

Given:

m = 54g = 5.4×10^{-2}kg

λ = 5400Å = 5400×10^{-10} m

v = ?

Formula used:

$$\lambda = \frac{h}{mv}$$

$$v = \frac{h}{m\lambda}$$

Solution:

$$v = \frac{h}{m\lambda}$$

$$v = \frac{6.626 \times 10^{-34}}{5.4 \times 10^{-2} \times 5400 \times 10^{-10}}$$

$v = 0.2238 \times 10^{-24}$ m/s

$$\boxed{v = 2.238 \times 10^{-25}\ m/s}$$

38. For each of the following, give the sub level designation, the allowable m values and the number of orbitals. i) n = 4, l =2, ii) n =5, l = 3 iii) n=7, l=0

n	l	Sub energy levels	m values	Number of orbitals
4	2	4d	-2, -1, 0, +1, +2	Five 4d orbitals
5	3	5f	-3, -2, -1, 0, +1, +2, +3	Seven 5f orbitals
7	0	7s	0	One 7s orbital

39. Give the electronic configuration of Mn^{2+} and Cr^{3+}.

- Mn (Z = 25)
  $Mn \rightarrow Mn^{2+} + 2e^-$
  Mn^{2+} electronic configuration is **1s $1s^2$ $2s^2$ $2p^6$ $3s^2$ $3p^6$ $3d^5$**

- Cr (Z = 24)
  $Cr \rightarrow Cr^{3+} + 3e^{-}$
  Cr^{3+} electronic configuration is $\mathbf{1s^2\ 2s^2\ 2p^6\ 3s^2 3p^6\ 3d^3}$

40. Describe the Aufbau principle.

- Aufbau principle states that, **'In the ground state of the atoms, the orbitals are filled in the order of their increasing energies.'**
- That is the electrons first occupy the lowest energy orbital available to them. Once the lower energy orbitals are completely filled, then the electrons enter the next higher energy orbitals.
- The order of filling of various orbitals as per Aufbau principle **(n+l rule)** is
  1 s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d …
- For e.g., K (Z =19)
  The electronic configuration is $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^1$.
  After **filling 4s** orbital only we have to **fill up 3d** orbital.

41. An atom of an element contains 35 electrons and 45 neutrons. Deduce
i) the number of protons
ii) the electronic configuration for the element
iii) All the four quantum numbers for the last electron

- No. of electrons = 35 (given)
  No. of protons = 35
- Electronic configuration:
  $\mathbf{1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^5}$
- The last electron i.e. **5^{th} electron** in 4p orbital has the following quantum numbers
  n = 4, l = 1, m =+1 or -1, s = +1/2

42. Show that the circumference of the Bohr orbit for the hydrogen atom is an integral multiple of the de Broglie wave length associated with the electron revolving around the nucleus.

Circumference of the orbit = $2\pi r \rightarrow 1$
Circumference of H-atom = $n\lambda \rightarrow 2$
$1 = 2 \Rightarrow 2\pi r = \lambda \Rightarrow \mathbf{2\pi r = \frac{h}{mv}}$

43. Calculate the energy required for the process.
$He^{+}(g) \rightarrow He^{2+}(g) + e^{-}$
The ionisation energy for the H atom in its ground state is 13.6 eV $atom^{-1}$.
Solution:

$E_n = \frac{-13.6z^2}{n^2}$

$E_1 = \frac{-13.6(2)^2}{1^2} = -56.4$

$E_\infty = \frac{-13.6(2)^2}{\infty^2} = 0$

Required energy = E_∞ - E_1
Required energy = 0 – (-56.4)

$\boxed{\textit{Required energy} = +56.4eV}$

44. An ion with mass number 37 possesses unit negative charge. If the ion contains 11.1% more neutrons than electrons. Find the symbol of the ion.
Solution:

Let x-1 be the number electrons and protons and y be the number of neutrons.
Given that, y = x + 11.1% of x
y = x + 0.111x
y = 1.111x
Mass number = 37
Number of protons + Number of neutrons = 37
(x - 1) + 1.111x = 37
2.111x = 38
$x = \frac{38}{2.11}$
x = 18.009 = 18(whole number)
Hence, Atomic number = x – 1 = 18 – 1 = 17
Mass number = 37

$\boxed{\textit{Symbol of the ion } {}^{37}_{17}Cl^{-}}$

45. The Li^{2+} ion is a hydrogen like ion that can be described by the Bohr model. Calculate the Bohr radius of the third orbit and calculate the energy of an electron in 4th orbit.
Solution:

$E_n = \frac{-13.6z^2}{n^2}$

$E_4 = \frac{-13.6(3)^2}{(4)^2}$

$\boxed{E_4 = -7.65\ eVatom^{-1}}$

$r_n = \frac{(0.529)n^2}{z}$

$r_3 = \frac{(0.529)(3)^2}{3}$

$\boxed{r_3 = 1.587\ Å}$

46. Protons can be accelerated in particle accelerators. Calculate the wavelength (in Å) of such accelerated proton moving at $2.85 \times 10^8\ ms^{-1}$ (the mass of proton is 1.673×10^{-27} Kg).
Given:
m = mass of the proton = 1.673×10^{-27} Kg
v = velocity of the proton = $2.85 \times 10^8\ ms^{-1}$
h = Planck's constant = $6.626 \times 10^{34}\ Kg\ m^2\ s^{-1}$
λ = ?
Formula used:

$$\lambda = \frac{h}{mv}$$

Solution:

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{1.673 \times 10^{-27} \times 2.85 \times 10^8}$$

$\lambda = 1.389 \times 10^{-15}$

$\boxed{\lambda = 1.389 \times 10^{-5}\ Å}$

47. What is the de Broglie wavelength (in cm) of a 160g cricket ball travelling at 140 Km/hr.
Given:

m = mass of the cricket ball = 160g = 0.16 kg.

v = velocity of the cricket ball =140 Km h^{-1} = $\frac{140 \times 10^3}{60 \times 60}$ = 38.88 ms^{-1}

λ = ?

Formula used:

$\lambda = \frac{h}{mv}$

Solution:

$\lambda = \frac{h}{mv}$

$\lambda = \frac{6.626 \times 10^{-34}}{0.16 \times 38.88}$

$\boxed{\lambda = 1.065 \times 10^{-34} m}$

48. Suppose that the uncertainty in determining the position of an electron in an orbit is 0.6 Å. What is the uncertainty in its momentum?

Given:

Δx = uncertainty in position of an electron = 0.6 Å = 0.6 x 10^{-10} m.

Δp = uncertainty in momentum = ?

Formula used:

$\Delta x . \Delta p \geq \frac{h}{4\pi}$

$\Delta p = \frac{h}{4\pi \Delta x}$

Solution:

$\Delta p = \frac{h}{4\pi \Delta x}$

$\Delta p = \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 0.6 \times 10^{-10}}$

$\boxed{\Delta p = 8.792 \times 10^{-25} kgms^{-1}}$

49. Show that if the measurement of the uncertainty in the location of the particle is equal to its de Broglie wavelength, the minimum uncertainty in its velocity (ΔV)is equal to 1/4π of its velocity(V).

Given:

$\Delta x = \lambda$

Δv = ?

Formula used:

$\Delta x . \Delta p \geq \frac{h}{4\pi}$

Solution:

$\Delta x . \Delta p \geq \frac{h}{4\pi}$

$\lambda . (m\Delta v) \geq \frac{h}{4\pi}$

$\Delta v \geq \frac{h}{4\pi \lambda m}$

$\Delta v \geq \frac{h}{4\pi \frac{h}{mv} m}$ $\quad (\lambda = \frac{h}{mv})$

$\boxed{\Delta v \geq \frac{v}{4\pi}}$

Therefore, minimum uncertainty in velocity = $\frac{v}{4\pi}$

50. What is the de Broglie wave length of an electron, which is accelerated from the rest, through a potential difference of 100V?

Given:

Potential difference = 100V = 100 × 1.6 × 10^{-19} J

λ = ?

Formula used:

$$\lambda = \frac{h}{\sqrt{2mev}}$$

Solution:

$$\lambda = \frac{h}{\sqrt{2mev}}$$

$$\lambda = \frac{6.626\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times100\times1.6\times10^{-19}}}$$

$\boxed{\lambda = 1.22 \times 10^{-10} \text{ m or } 1.22 \text{ Å}}$

51. Identify the missing quantum numbers and the sub energy level.

n	l	m	Sub energy level
4	**2**	0	4d
3	1	0	**3p**
5	**1**	**-1 or 0 or +1**	5p
3	**2**	-2	3d