One two academy

Prepare - Pray - Perform
Volume 1 Important Questions
CHAPTER 1

Two Marks:-

1)Prove that $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ is orthogonal.

2)Find the rank of the matrix $\begin{bmatrix} 4 & -2 & 6 & 8 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{bmatrix}$.

3)Find the rank of the matrix $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$.

4)If adj(A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$. Find A^{-1}.

5)If A is a non singular matrix of odd order, prove that |adjA| is positive.

6) If A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ B = $\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$, find adj(AB).

Three marks:-

1)Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$.

2)Solve the following system of linear equations by matrix inversion method
2x - y = 8 ; 3x + 2y = -2.

3)Solve by matrix inversion method 5x + 2y = 4, 7x + 3y = 5.

4)If A = $\begin{bmatrix} 1 & 3 \\ 2 & -5 \end{bmatrix}$ verify that A(adjA) = (adjA)A = |A|I.

5)Verify the property $(A^T)^{-1} = (A^{-1})^T$ with A = $\begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

6)If A = $\begin{bmatrix} 3 & -2 \\ a & -2 \end{bmatrix}$, find a so that $A^2 = aA - 2I$.

7)Find the rank of the matrix $\begin{bmatrix} 4 & -2 & 6 & 8 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{bmatrix}$.

Five marks:-

1)If $ax^2 + bx + c$ is divided by x + 3, x - 5 and x -1, the remainders are 21, 61 and 9 respectively. Find a, b and c.

2) A = $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

3)A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadaisis ₹150. The cost of the two dosai, two idlies and four vadais is ₹200. The cost of five dosai, four idlies and two vadais is ₹250. The family has ₹350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

4)Examine the consistency of the system of equations 4x + 3y + 6z = 25, x + 5y + 7z = 13 and 2x + 9y + z = 1. If it is consistent then solve.

5)Investigate for what values of λ and μ the system of linear equation x + 2y + z = 7, x + y + λz = μ, x +3y -5z = 5 (i) no solution (ii) a unique solution.

6)Determine the values of λ for which the system of linear equations x + y + 3z =0, 4x + 3y + λz =0, 2x + y + 2z =0. Has (i) unique solution (ii) a non- trivial solution.

7)Test the consistency and if possible solve the equation 2x-y+z=2, 6x-3y+3z=6, 4x-2y+2z=4.

8)Find the inverse of the non-singular matrix $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by elementary transformation.

CHAPTER 2

Two marks:-

1)Find the modulus of $\frac{1-i}{3+i} + \frac{4i}{5}$.

2)Find z^{-1}, if z = (2+3i)(1-i)

3)Find the square root of -6 + 8i.

4)Show that $(2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$ is real.

5)If z1 = 2-i and z2 = -4 +3i, find the inverse of $\frac{z1}{z2}$.

6) $\sum_{i=1}^{12} i^n$ Simplify

7)Show that the equation |2z + 2 − 4i| = 2represent a circle, and find its centre and radius.

8)Show that $(2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$ is real.

9) If x + iy = $\sqrt{\frac{a+ib}{c+id}}$, prove that $x^2 + y^2 = \sqrt{\frac{a^2+b^2}{c^2+d^2}}$.

10.Express $-1+i\sqrt{3}$ in polar form.

Three marks:-

1)Show that the equation has $z^3 + 2\check{z} = 0$ has five solution.

2)Show that the points 1, $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$ and $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of the equilateral triangle.

3)Show that the equation |2z + 2 − 4i| = 2represent a circle, and find its centre and radius.

4)Find the value of $\sum_{k=1}^{8} cos\frac{2k\pi}{9} + isin\frac{2k\pi}{9}$.

5)Find the value of $\frac{cos\frac{\pi}{6} + isin\frac{\pi}{6}}{cos\frac{\pi}{3} - isin\frac{\pi}{3}}$.

6)If $\omega \neq 1$ is a cube root of unity and $(1+\omega)^7 = A + B\omega$. Then find A and B.

Five marks:-

1)If z_1, z_2 and z_3 are three complex numbers such that $|z_1| = 1$. $|z_2| = 2$, $|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that $|9z_1z_2 + 4z_1z_3 + z_2z_3|$.

2)Show that $[\frac{19-7i}{9+i}]^{12} + [\frac{20-5i}{7+6i}]^{12}$ is real.

3)If z = x+iy and arg(z-i/z+2) = $\pi/4$, then show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

4)Solve $z^4 = 1-\sqrt{3}i$.

5)Simplify: $(-\sqrt{3}+3i)^{31}$.

6)$\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ then show that

(i)$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\beta)$.

(ii)$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\beta)$.

7)Solve $z^3 + 8i = 0$l where $z \in \mathbf{C}$.

8)Find all the cube roots of unity $\sqrt{3}+i$.

9)If z = x+iy and arg(z-1/z+1) = $\pi/2$, then show that $x^2 + y^2 = 1$.

CHAPTER 3

Two marks:-

1)Find the polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}i$ as a root.
2)Find the exact number of real zeros and imaginary roots of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x = 0$.

Three marks:-

1)Solve $x^3 - 5x^2 - 4x + 20 = 0$.
2)Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has at least 6 imaginary solutions.
3)If α and β are the roots of the equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.
4)If the sides of a cubic box are increased by 1,2,3 units respectively to form a cuboid , then the volume increased-by 52 cubic units . Find the volume of the cuboid.

Five mark:-

1)Solve the equation:- $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$.
2)Find the sum of squares of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$.
3)Solve $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.
4)Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.
5)Find a polynomial equation of minimum degree with rational coefficients having $\sqrt{5} - \sqrt{3}$ as a root.

CHAPTER 4

Two marks:-

1)Find the value of $\tan^{-1}(\tan(\frac{-\pi}{6}))$.
2)Find the value of $\sin^{-1}[\sin[\frac{5\pi}{4}]]$.
3)Write the principle value of $\tan^{-1}[\sin(-\frac{\pi}{2})]$.
4)State the reason for $\cos^{-1}(\cos(-\frac{\pi}{6}))$ =/= $-\frac{\pi}{6}$.
5)Find the value of $\cos^{-1}(\frac{1}{2}) - 2\sin^{-1}\frac{1}{2}$.
6)Find the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$.
7) For what value of x does $\sin x = \sin^{-1} x$.?
8)Find the principal value of $\sec^{-1}(-2)$

Three marks:-

1)For what value of x, this inequality $\frac{\pi}{2} < cos^{-1}(3x - 1) < \pi$ hold ?
2)Sketch the graph of $y = \sin \frac{1}{3}x$ for $0 <= x <= 6\pi$.

Five marks:-

1)Find the value of $\cos(\sin^{-1}\frac{4}{5} \tan^{-1}\frac{3}{4})$.
2)Solve for x: $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$.
3)Solve $\cos(\tan^{-1}x) = \sin(\cot^{-1}\frac{3}{4})$.
4)Evaluate $\sin[\sin^{-1}3/5 + \sec^{-1}5/4]$.
5)Draw the curve sinX in the domain $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and $\sin^{-1}$ in [-1,1].
6)Solve for x :- $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, if $6x^2 < 1$.

CHAPTER 5

Two marks:-

1)Find the centre and radius of the circle $3x^2 + 3y^2 - 12x + 6y - 9 = 0$.
2)Obtain the equation of the circle for which (3,4) and (2,-7) are the ends of a diameter.
3))Find the length of the latus rectum if the hyperbola $16y^2 - 9x^2 = 144$.
4))Find the centre and radius of the circle $2x^2 + 2y^2 - 6x + 4y + 2 = 0$.
5)Find the equation of the parabola whose end points of the latus rectum are (4,-8) and (4,8) and centre is (0,0) and open rightward.

6)Identify the type of conic $y^2 + 4x + 3y + 4 = 0$.

Three marks:-

1)If the equation $3x^2 + (3-p)xy + qy^2 -2px = 8pq$ represents a circle. Find p and q. Also determine the centre and radius of the circle.

2)Prove that the length of latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $2\frac{b^2}{a}$.

3)Find the equation of the hyperbola with foci (+/-3,5) and eccentricity e = 2.

4)Find the foci of $9x^2 - 16y^2 = 144$.

5)The line 3x +4y -12 =0 meets the coordinate axes at A and B . Find the equation of the circle drawn on AB as diameter.

6)The maximum and minimum distance of the earth from the sun respectively are 152×10^6km and 94.5×10^6 km. The sun is at one focus of the elliptical orbit. From the distance from the sun to other focus.

7)Find the equation of the hyperbola with vertices (0,+/-4) and foci (0,+/-6).

8)Find the equation of the circle with centre (2,3) and passing through the intersection of the lines $3x-2y-1=0$and $4x+y-27=0$

9)Find the equation of tangent to the curve $x^2 y - x = y^3 -8$ at x = 0.

Five marks:-

1)At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5m from its origin. The flow is from the origin and the path of water is a parabola open upwards, find the height of water at a horizontal distance of 0.75m from the point of origin.

2)Examine the hyperbola $4x^2 -24x - 25y^2 + 250y - 489 = 0$.

3))For the ellipse $4x^2 + y^2 +24x -2y + 21 = 0$, find centre, vertices, foci and the length of latus rectum.

4)The eccentricity of an ellipse with its centre at the origin is 1/2. If one of the directrix is x = 4, then find the equation of the ellipse.

5)Find the equation of the circle through the points (1,0), (-1,0) and (0,1).

6)On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

7)A rod of length 1 2. m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0 3. m from the end in contact with x -axis is an ellipse. Find the eccentricity.

8)Assume that water issuing from the end of a horizontal pipe, 7 5. m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2 5. m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

9)An elliptical whispering room has height 5m and width 26m. Where should the two persons stand if they would like to whisper back and forth and be heard.

CHAPTER 6

1)Verify whether the line $\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{12}$ lies in the plane 5x - y + z = 8.

2)Find the intercepts cut off by the plane $\vec{r}.(6\vec{i} + 4\vec{j} - 3\vec{k}) = 12$ on coordinate axes.

3)Find the angle between the lines 2x = 3y = -z and 6x = -y =-4z.

4)Find the vector equation of the plane passing through the point (2,2,3) having 3,4,3 as direction ratios of the normal to the plane.

5)Find the angle between straight lines $\vec{r}$ = ***(2i+3j+k)*** + t(***i-j+k)*** and the plane 2x -y + z = 5.

Three marks:-

1) If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors prove that $[\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}] = 2\,[\vec{a}\,.\,\vec{b}, \vec{c}]$.
2) Find the altitude of a parallelepiped determined by the vectors $\vec{a} = -2\vec{i} + 5\vec{j} + 3\vec{k}$, $\vec{b} = \vec{i} + 3\vec{j} - 2\vec{k}$ and $\vec{c} = -3\vec{i} + \vec{j} + 4\vec{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ and $\vec{c}$.
3) Find the cartesian equation of the plane passing through the lime of intersection of the planes $\vec{r}.(2\vec{i} - 7\vec{j} + 4\vec{k}) = 3$ and 3x - 5y + 4z + 11 = 0 and the point (-2,1,3).
4) Find the magnitude and d.c.s of the moment about the point (0,-2,3) of a force ***i+j+k*** whose line of action passing through the origin.

5) Find the magnitude and d.c.s of the torque of a force represented by 3***i* + *4j -5k*** about the point with position vector ***2i - 3j + 4k*** acting through a point whose position vector is ***4i + 2j -3k***.

6)If $\vec{a}$ = ***i+2j+3k*** , $\vec{b}$ = ***2i-j+k,*** $\vec{c}$= ***3i+2j+k*** and $\vec{a}x(\vec{b}x\vec{c}) = l\vec{a} + m\vec{b} + n\vec{c}$. Find the values of l, m and n.

7)If two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-m}{2} = z$ intersect at a point , find m.

Five marks:

1) If $\vec{a} = \vec{i} - \vec{j}, \vec{b} = \vec{i} - \vec{j}$, $\vec{c} = 3\vec{j} - \vec{k}$ and $\vec{d} = 2\vec{i} + 5\vec{j} + \vec{k}$, verify that $(\vec{a}x\vec{b})x(\vec{c}x\vec{d})$ = $[\vec{a}, \vec{b}, \vec{d}]\vec{c}$ - $[\vec{a}, \vec{b}, \vec{c}]\vec{d}$.
2) Find the vector and cartesian equation of the plane. $\vec{r} = (6\vec{i} - \vec{j} + \vec{k})$ + s$(-\vec{i} + 2\vec{j} + \vec{k})$ + t$(-5\vec{i} - 4\vec{j} - 5\vec{k})$.
3) Find the vector and cartesian equation of the plane passing through the point (1,-2,4) and perpendicular to the plane x + 2y -3z = 11 and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$
4) Find the foot of the perpendicular drawn from the point (5,4,2) to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$. Also find the equations of the perpendicular.
5) Using vector method derive the formula for $\cos(\alpha + \beta)$.
6) Find the shortest distance between the straight lines $\frac{x-6}{1} = \frac{2-y}{2} = \frac{z-2}{2}$ and $\frac{x+4}{3} = \frac{y}{-2} = \frac{1-z}{2}$.
7) If the straight lines $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{\lambda}$ are coplanar , find λ and cartesian equation of the planes containing these two lines.
8) If $\vec{a}$ = ***i+j-k*** $\vec{b}$ = ***2i + 3j*** $\vec{c}$ = ***j-k*** verify $\vec{a}x(\vec{b}x\vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$.
9) State and prove Apollonius theorem.
10) Show that the straight lines $\vec{r}$ = ***(5i + 7j -3k)*** + s***(4i+4j-5k)*** and $\vec{r}$ = ***(8i+4j+5k)*** + t***(7i+j+3k)*** are Coplanar. Find the vector equation of the plane in which they lie.
11) Show that the points (6,-7,0) , (16,-19,-4) , (0,3,-6) , (2,-5,10) lie on a same plane.
12) Show that the straight lines x+1 = 2y = -12z and x = y + 2 = 6z -6 are skew and hence find the shortest distance between them.