ONE TWO ACADEMY **UNIT TEST**

MATHEMATICS **HSC 1ST YEAR**

SECTION - A

Answer the following:- **(any 3) Question no 5 is compulsory** **3 x 5 = 15**

1) Prove that the medians of triangle are concurrent.

2) Prove that the line segment joining the midpoints of two sides of a triangle is parallel to the
third side whose length is half of the length of the third side.

3) Prove that the line segments joining the midpoints of the adjacent sides of a quadrilateral
form a parallelogram.

4) Prove that the points whose position vectors $2\hat{i}+4\hat{j}+3\hat{k}$, $4\hat{i}+\hat{j}+9\hat{k}$ and $10\hat{i}-\hat{j}+6\hat{k}$ form a right angled triangle.

5) Show that the points whose position vectors $4\hat{i}+5\hat{j}+\hat{k}$, $-\hat{j}-\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $-4\hat{i}+4\hat{j}+4\hat{k}$ are coplanar.

6) The position vectors of the vertices of a triangle are $\hat{i}+2\hat{j}+3\hat{k}$; $3\hat{i}-4\hat{j}+5\hat{k}$ and $-2\hat{i}+3\hat{j}-7\hat{k}$. Find the perimeter of the triangle.

SECTION - B

Answer the following :(**any 5) Question no 14 is compulsory** **5 x 3 = 15**

8) If ABC and A′B′C′ are two triangles and G, G′ be their corresponding centroids, prove that $\overrightarrow{AA'} + \overrightarrow{BB'} + \overrightarrow{CC'} = 3\overrightarrow{GG'}$

9) For any vector $\vec{r}$ prove that $\vec{r} = (\vec{r}\cdot\hat{i})\hat{i} + (\vec{r}\cdot\hat{j})\hat{j} + (\vec{r}\cdot\hat{k})\hat{k}$.

10) A triangle is formed by joining the points (1, 0, 0), (0, 1, 0) and (0, 0, 1). Find the direction cosines of the medians.

11) Prove that the relation R defined on the set V of all vectors by '$\vec{a}\ R\ \vec{b}$ if $\vec{a} = \vec{b}$' is an equivalence relation on V.

12) Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$, $|\vec{c}| = 4$, and $\vec{a}+\vec{b}+\vec{c} = \vec{0}$. Find $4\vec{a}\cdot\vec{b} + 3\vec{b}\cdot\vec{c} + 3\vec{c}\cdot\vec{a}$.

13) Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$.

Prove that $\vec{a} = \pm\frac{2}{\sqrt{3}}(\vec{b}\times\vec{c})$.

14) If $\vec{a}, \vec{b}$ are unit vectors and θ is the angle between them, show that $\tan\frac{\theta}{2} = \frac{|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|}$.

SECTION - C

Answer the following:- **(any 5) Question no 7 is compulsory** **5 x 2 = 10**

1) If $\vec{a}, \vec{b}$ and $\vec{c}$ are the sides of a triangle taken in order then $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ Prove

2) Prove that sum of direction sines is 2

3) Find the angle between the vectors

(i) $2\hat{i}+3\hat{j}-6\hat{k}$ and $6\hat{i}-3\hat{j}+2\hat{k}$

4) If $\frac{1}{2}, \frac{1}{\sqrt{2}}, a$ are the direction cosines of some vector, then find a.

5) Find the projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ where A, B, C, D are the points (4, – 3, 0), (7, – 5, – 1), – 2, 1, 3), (0, 2, 5).

6) Find the value or values of m for which $m\,(\hat{i}+\hat{j}+\hat{k})$ is a unit vector.

7) For any two vectors $\vec{a}$ and $\vec{b}$, prove that $|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2\ |\vec{b}|^2$

SECTION - D

Choose the correct answer :- 10 x 2 = 20

IMPORTANT NOTE:- Solution for each one markers has to be stated neatly in the answer sheet. (Correct answer WO solution - 1 | Correct answer with solution - 2)

1) If $|\vec{a}|=13, |\vec{b}|=5$ and $\vec{a}\cdot\vec{b}=60°$ then $|\vec{a}\times\vec{b}|$ is

(1) 15 (2) 35 (3) 45 (4) 25

2) Vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle $\theta=120°$. If $|\vec{a}|=1, |b|=2$, then $[(\vec{a}+3\vec{b})\times(3\vec{a}-\vec{b})]^2$ is equal to

(1) 225 (2) 275 (3) 325 (4) 300

3) If $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 2 and inclined at an angle $60°$, then the angle between $\vec{a}$ and $\vec{a}+\vec{b}$ is

(1) 30° (2) 60° (3) 45° (4) 90°

4) If the projection of $5\hat{i}-\hat{j}-3\hat{k}$ on the vector $\hat{i}+3\hat{j}+\lambda\hat{k}$ is same as the projection of $\hat{i}+3\hat{j}+\lambda\hat{k}$ on $5\hat{i}-\hat{j}-3\hat{k}$, then λ is equal to

(1) ± 4 (2) ± 3 (3) ± 5 (4) ± 1

5) If (1, 2, 4) and $(2, -3\lambda-3)$ are the initial and terminal points of the vector $\hat{i}+5\hat{j}-7\hat{k}$, then the value of λ is equal to

(1) $\frac{7}{3}$ (2) $-\frac{7}{3}$ (3) $-\frac{5}{3}$ (4) $\frac{5}{3}$

6) If the points whose position vectors $10\hat{i}+3\hat{j}$, $12\hat{i}-5\hat{j}$ and $a\hat{i}+11\hat{j}$ are collinear then a is equal to

(1) 6 (2) 3 (3) 5 (4) 8

7) If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+x\hat{j}+\hat{k}$, $\vec{c}=\hat{i}-\hat{j}+4\hat{k}$ and $\vec{a}\cdot(\vec{b}\times\vec{c})=70$, then x is equal to

(1) 5 (2) 7 (3) 26 (4) 10

8) If $\vec{a}=\hat{i}+2\hat{j}+2\hat{k}, |\vec{b}|=5$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then the area of the triangle formed by these two vectors as two sides, is

(1) $\frac{7}{4}$ (2) $\frac{15}{4}$ (3) $\frac{3}{4}$ (4) $\frac{17}{4}$

9) If $\lambda\hat{i}+2\lambda\hat{j}+2\lambda\hat{k}$ is a unit vector, then the value of λ is

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{1}{9}$ (4) $\frac{1}{2}$

10) Two vertices of a triangle have position vectors $3\hat{i}+4\hat{j}-4\hat{k}$ and $2\hat{i}+3\hat{j}+4\hat{k}$. If the position vector of the centroid is $\hat{i}+2\hat{j}+3\hat{k}$, then the position vector of the third vertex is

(1) $-2\hat{i}-\hat{j}+9\hat{k}$ (2) $-2\hat{i}-\hat{j}-6\hat{k}$ (3) $2\hat{i}-\hat{j}+6\hat{k}$ (4) $-2\hat{i}+\hat{j}+6\hat{k}$

SECTION - E

Choose the correct answer:- 10 x 1 = 10

1) If $\overrightarrow{AB} = k\ \overrightarrow{AC}$ where k is a scalar then

(1) A, B, C are collinear (2) A, B, C are coplanar

(3) $\overrightarrow{AB}$, $\overrightarrow{AC}$ have the same magnitude (4) A, B, C are coincident

2 The position vectors of A and B are $\vec{a}$ and $\vec{b}$. P divides AB in the ratio 3 : 1. Q is the mid point of AP. The position vector of Q is

(1) $\frac{5\vec{a} + 3\vec{b}}{8}$ (2) $\frac{3\vec{a} + 5\vec{b}}{2}$ (3) $\frac{5\vec{a} + 3\vec{b}}{4}$ (4) $\frac{3\vec{a} + \vec{b}}{4}$

3) If G is the centriod of a triangle ABC and O is any other point then $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$ is equal to

(1) $\vec{O}$ (2) $\overrightarrow{OG}$ (3) $3\ \overrightarrow{OG}$ (4) $4\ \overrightarrow{OG}$

4) If G is the centriod of a triangle ABC then $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC}$ is equal to

(1) $3\left(\vec{a} + \vec{b} + \vec{c}\right)$ (2) $\overrightarrow{OG}$ (3) $\vec{O}$ (4) $\frac{\vec{a} + \vec{b} + \vec{c}}{3}$

5) If G is the centriod of a triangle ABC and G′ is the centroid of triangle A′ B′ C′ then $\overrightarrow{AA'} + \overrightarrow{BB'} + \overrightarrow{CC'} =$

(1) $\overrightarrow{GG'}$ (2) $3\overrightarrow{GG'}$ (3) $2\overrightarrow{GG'}$ (4) $4\overrightarrow{GG'}$

6) If $\vec{a}$ is a non-zero vector and m is a non-zero scalar then $m\vec{a}$ is a unit vector if

(1) $m = \pm 1$ (2) $a = |m|$ (3) $a = \frac{1}{|m|}$ (4) $a = 1$

7) If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is the angle between them, then $\left(\vec{a} + \vec{b}\right)$ is a unit vector if

(1) $\theta = \frac{\pi}{3}$ (2) $\theta = \frac{\pi}{4}$ (3) $\theta = \frac{\pi}{2}$ (4) $\theta = \frac{2\pi}{3}$

8) If $\vec{a}$ and $\vec{b}$ include an angle $120°$ and their magnitude are 2 and $\sqrt{3}$ then $\vec{a} \,.\, \vec{b}$ is equal to

(1) $\sqrt{3}$ (2) $-\sqrt{3}$ (3) 2 (4) $-\frac{\sqrt{3}}{2}$

9) If $\vec{u} = \vec{a} \times \left(\vec{b} \times \vec{c}\right) + \vec{b} \times \left(\vec{c} \times \vec{a}\right) + \vec{c} \times \left(\vec{a} \times \vec{b}\right)$, then

(1) u is a unit vector (2) $\vec{u} = \vec{a} + \vec{b} + \vec{c}$

(3) $\vec{u} = \vec{0}$ (4) $\vec{u} \neq \vec{0}$

10) The area of the parallelogram having a diagonal $3\vec{i} + \vec{j} - \vec{k}$ and a side $\vec{i} - 3\vec{j} + 4\vec{k}$ is

(1) $10\sqrt{3}$ (2) $6\sqrt{30}$ (3) $\frac{3}{2}\sqrt{30}$ (4) $3\sqrt{30}$