One Two academy | Discrete Mathematics

Maximum Mark: 25 **Mathematics** **Time: 50 minutes**

Choose the correct answer: **3 x 1 = 3**

1. In the multiplicative group of nth roots of unity, the inverse of ω^k is $(k < n)$

(a) $\omega^{\frac{1}{k}}$ (b) ω^{-1} (c) ω^{n-k} d) $\omega^{\frac{n}{k}}$

2. If a compound statement involves 3 simple statements, then the number of rows in the truth table is

(a) 9 (b) 8 (c) 6 (d) 3

3. The dual of ¬ (pvq) v [pv(p∧¬r)] is

(1) ¬ (p∧q) ∧ [pv(p∧¬r)] (2) (p∧q) ∧ [p∧ (pv¬r)]

(3) ¬ (p∧q) ∧ [p∧ (p∧r)] (4) ¬ (p∧q) ∧ [p∧ (pv¬r)]

Answer the following: **3 x 2 = 6**

4. Let * be defined on R by (a*b) = a + b + ab – 7. Is * binary on R? If so, find $3 * \left(-\frac{7}{15}\right)$.

5. From a conditional statement $p \to q$, list out the conditional statements which can be derived.

6. What is the condition for two statements to be logically equivalent?

Answer any two of the following: **2 x 3 = 6**

7.

Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ be any three boolean matrices

of the same type. Find (i) $A \vee B$ (ii) $A \wedge B$ (iii) $(A \vee B) \wedge C$ (iv) $(A \wedge B) \vee C$.

8. State and prove De-Morgan's law. (any one)

9. Determine the truth value of each of the following statements

(i) If 6+2= 5 , then the milk is white.

(ii) China is in Europe or 3 is an integer

(iii) It is not true that 5+5=9 or Earth is a planet

(iv) 11 is a prime number and all the sides of a rectangle are equal

Answer the following: **2 x 5 = 10**

10. Prove that the set of four operations f_1, f_2, f_3, f_4 on the set of non-zero complex number $C - \{0\}$ defined by

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$$f_1(z) = z, f_2(z) = -z, f_3(z) = \frac{1}{z}, f_4(z) = -\frac{1}{z}$$

$\forall\, z \in C - \{0\}$ forms an abelian group with respect to the composition of functions.

Hint: *An abelian group satisfies closure, commutative, associative, identity and inverse axioms are satisfied.*

11. Construct the truth table for

$$(p \bar{\vee} q) \wedge (p \bar{\vee} \neg q)$$

OR

Using equivalence property, show that

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q).$$

All the Best

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Answer Key attached in the next page !!!

This page is to prevent seeing answer key

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Answer Key

<table>
<tr><th>Q.no</th><th>Solution</th><th>Marks</th></tr>
<tr><td>1.</td><td>(c) ω^{n-k}</td><td>1</td></tr>
<tr><td>2.</td><td>(b) 8</td><td>1</td></tr>
<tr><td>3.</td><td>¬ (p∧q) ∧ [p∧ (pv¬r)] (d)</td><td>1</td></tr>
<tr><td>4.</td><td>$\frac{-88}{15}$</td><td>2</td></tr>
<tr><td>5.</td><td>(i) Converse statement $q \to p$.
(ii) Inverse statement $\neg p \to \neg q$.
(iii) Contrapositive statement $\neg q \to \neg p$.</td><td>2</td></tr>
<tr><td>6.</td><td>$A \longleftrightarrow$ B is a tautology.</td><td>2</td></tr>
<tr><td>7.</td><td>(i) $A \vee B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ (ii) $A \wedge B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$
(iii) $(A \vee B) \wedge C = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ (iv) $(A \wedge B) \vee C = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$</td><td>3</td></tr>
<tr><td>8.</td><td>(i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (ii) $\neg(p \vee q) \equiv \neg p \wedge \neg q$
Proof of (i)
<table>
<tr><th>p</th><th>q</th><th>$\neg p$</th><th>$\neg q$</th><th>$p \wedge q$</th><th>$\neg(p \wedge q)$</th><th>$\neg p \vee \neg q$</th></tr>
<tr><td>T</td><td>T</td><td>F</td><td>F</td><td>T</td><td>F</td><td>F</td></tr>
<tr><td>T</td><td>F</td><td>F</td><td>T</td><td>F</td><td>T</td><td>T</td></tr>
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</table>
The entries in the columns corresponding to $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are identical and hence they are equivalent. Therefore $\neg(p \wedge q) \equiv \neg p \vee \neg q$. Dually (ii) $\neg(p \vee q) \equiv \neg p \wedge \neg q$ can be proved.</td><td>3</td></tr>
<tr><td>9.</td><td>(i) $p \to q$ is $\mathbb{T}$ (ii) $p \vee q$ is $\mathbb{F}$ (iii) $\neg p \vee q$ is $\mathbb{T}$ (iv) $p \wedge q$ is $\mathbb{F}$</td><td>3</td></tr>
</table>

10. **Solution:** Let $G = \{f_1, f_2, f_3, f_4\}$ **5**

$(f_1 \circ f_1)(z) = f_1(f_1(z)) = f_1(z)$

$\therefore f_1 \circ f_1 = f_1$

$f_2 \circ f_1 = f_2$, $f_3 \circ f_1 = f_3$, $f_4 \circ f_1 = f_4$

Again $(f_2 \circ f_2)(z) = f_2(f_2(z)) = f_2(-z) = -(-z) = z = f_1(z)$

$\therefore f_2 \circ f_2 = f_1$

Similarly $f_2 \circ f_3 = f_4$, $f_2 \circ f_4 = f_3$

$(f_3 \circ f_2)(z) = f_3(f_2(z)) = f_3(-z) = -\frac{1}{z} = f_4(z)$

$\therefore f_3 \circ f_2 = f_4$

Similarly $f_3 \circ f_3 = f_1$, $f_3 \circ f_4 = f_2$

$(f_4 \circ f_2)(z) = f_4(f_2(z)) = f_4(-z) = -\frac{1}{-z} = \frac{1}{z} = f_3(z)$

$\therefore f_4 \circ f_2 = f_3$

Similarly $f_4 \circ f_3 = f_2$, $f_4 \circ f_4 = f_1$

Using these results we have the composition table as given below :

$\circ$	f_1	f_2	f_3	f_4
f_1	f_1	f_2	f_3	f_4
f_2	f_2	f_1	f_4	f_3
f_3	f_3	f_4	f_1	f_2
f_4	f_4	f_3	f_2	f_1

From the table

(i) All the entries of the composition table are the elements of G .

$\therefore$ Closure axiom is true.

(ii) Composition of functions is in general associative.

(iii) Clearly f_1 is the identity element of G and satisfies the identity axiom.

(iv) From the table :

Inverse of f_1 is f_1 ; Inverse of f_2 is f_2

Inverse of f_3 is f_3 ; Inverse of f_4 is f_4

Inverse axiom is satisfied. (G, o) is a group.

(v) From the table the commutative property is also true.

$\therefore$ (G, o) is an abelian group.

11. **5**

p	q	$\neg q$	r : $(p \bar{\vee} q)$	s: $(p \bar{\vee} \neg q)$	$r \wedge s$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	F

OR

It can be obtained by using examples 12.17 and 12.18 that

$$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p) \quad \text{... (1)}$$

$$\equiv (\neg p \vee q) \wedge (p \vee \neg q) \text{ (by Commutative Law)} \quad \text{... (2)}$$

$$\equiv (\neg p \wedge (p \vee \neg q)) \vee (q \wedge (p \vee \neg q)) \text{ (by Distributive Law)}$$

$$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee (q \wedge \neg q) \text{ (by Distributive Law)}$$

$$\equiv \mathbb{F} \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee \mathbb{F}; \text{ (by Complement Law)}$$

$$\equiv (\neg p \wedge \neg q) \vee (q \wedge p); \text{ (by Identity Law)}$$

$$\equiv (p \wedge q) \vee (\neg p \wedge \neg q); \text{ (by Commutative Law)}$$

Finally (1) becomes $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.

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