One Two academy | Probability Distribution

Maximum Mark: 25 **Mathematics** **Time: 50 minutes**

Choose the correct answer: **3 x 1 = 3**

1. Given E(X+c) = 8 and E(X-c) = 12, then the value of c is 131

(a) -2 (b) 4 (c) -4 (d) 2

2. The mean of a binomial distribution is 5 and it's standard deviation is 2. Then the value of n and p are 137

(a) $(\frac{4}{5}, 25)$ (b) $\left(25, \frac{4}{5}\right)$ (c) $\left(\frac{1}{5}, 25\right)$ (d) $\left(25, \frac{1}{5}\right)$

3. Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with Probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result. The value of E(X) is

(a) 0.11 (b) 1.1 (c) 11 (d) 1

Answer the following: **3 x 2 = 6**

4. The difference between the mean and the variance of a Binomial distribution is 1 and the difference between their squares is 11. Find n.

5. If X is the random variable with distribution function F(x) given by,

$$F(x) = \begin{cases} 0, & -\infty < x < 0 \\ \frac{1}{2}\left(x^2 + x\right) & 0 \le x < 1 \\ 1, & 1 \le x < \infty \end{cases}$$

Then find (i) the probability density function (ii) $P(0.3 \le X \le 0.6)$.

6. Prove that $Var(X) = E(X^2) - (E(X))^2$.

Answer any two of the following: **2 x 3 = 6**

7. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win Rs 15 for each red ball selected and we lose Rs 10 for each black ball selected. If X denotes the winning amount, then find the values of X and the number of points in its inverse images.

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8. The cumulative distribution of a discrete random variable is given by

$$F(x)=\begin{cases}0 & \text{for } -\infty < x < 0\\ \frac{1}{2} & \text{for } 0 \le x < 1\\ \frac{3}{5} & \text{for } 1 \le x < 2\\ \frac{4}{5} & \text{for } 2 \le x < 3\\ \frac{9}{10} & \text{for } 3 \le x < 4\\ 1 & \text{for } 4 \le x < \infty\end{cases}$$

Find (i) the probability mass function (ii) P(X<3) (iii) $P(X \geq 2)$

9. The probability that Mr. Q hits a target at any trial is $\frac{1}{4}$. Suppose he tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) atleast one time.

Answer the following: **2 x 5 = 10**

10. Suppose that f(x) given below represents a pmf

x	1	2	3	4	5	6
f(x)	c^2	$2c^2$	$3c^2$	$4c^2$	c	2c

Find (i) the value of c (ii) Mean and Variance

OR

The probability density function of the random variable X is given by

$$f(x)=\begin{cases}16\,x\,e^{-4x} & \text{for } x>0\\ 0 & \text{for } x\le 0\end{cases}$$

Find the mean and variance of X.

11. Let X be a random variable denoting the life time of an electrical equipment having pdf

$$\text{f(x)} = \begin{cases}k\,e^{-2x}, & x>0\\ 0, & x\le 0\end{cases}$$

Find (i) the value of k. (ii) Distribution function (iii) $P(X=3)$.

All the Best

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Answer Key attached in the next page !!!

This page is to prevent seeing answer key

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Answer Key

<table>
<tr><th>Q.no</th><th>Solution</th><th>Marks</th></tr>
<tr><td>1.</td><td>(a) -2</td><td>1</td></tr>
<tr><td>2.</td><td>(d) $\left(25, \frac{1}{5}\right)$</td><td>1</td></tr>
<tr><td>3.</td><td>(b) 1.1</td><td>1</td></tr>
<tr><td>4.</td><td>Solution : Let the mean be $(m + 1)$ and the variance be m from the given data.[Since mean > variance in a binomial distribution]
$(m+1)^2 - m^2 = 11 \Rightarrow m = 5$
$\therefore$ mean $= m + 1 = 6$
$\Rightarrow np = 6$; $npq = 5$ $\therefore q = \frac{5}{6}$, $p = \frac{1}{6}$ $\Rightarrow n = 36$.</td><td>2</td></tr>
<tr><td>5.</td><td>(i)
$f(x) = \begin{cases} x + \frac{1}{2}, & 0 \le x < 1 \\ 0, & otherwise \end{cases}$
(ii) 0.285</td><td>2</td></tr>
<tr><td>6.</td><td>(ii) $Var(X) = E(X^2) - (E(X))^2$
Proof
We know $E(x) = \mu$
$Var(X) = E(X-\mu)^2$
$= E(X^2 - 2X\mu + \mu^2)$
$= E(X^2) - 2\mu E(X) + \mu^2$ (Since μ is a constant)
$= E(X^2) - 2\mu\mu + \mu^2 = E(X^2) - \mu^2$
$Var(X) = E(X^2) - (E(X))^2$
An alternative formula to compute variance of a random variable X is
$\sigma^2 = \text{Var}(X) = E(X^2) - (E(X))^2$</td><td>2</td></tr>
<tr><td>7.</td><td><table><tr><td>Values of the random variable</td><td>-20</td><td>5</td><td>30</td><td>Total</td></tr><tr><td>Number of elements in inverse image</td><td>28</td><td>48</td><td>15</td><td>91</td></tr></table></td><td>3</td></tr>
<tr><td>8.</td><td><table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td></td><td>$\frac{1}{2}$</td><td>$\frac{1}{10}$</td><td>$\frac{1}{5}$</td><td>$\frac{1}{10}$</td><td>$\frac{1}{10}$</td></tr></table>(ii) $P(X < 3) = \frac{4}{5}$
(iii) $P(x \ge 2) = \frac{2}{5}$</td><td>3</td></tr>
<tr><td>9.</td><td>(i) $10C_4 \left(\frac{1}{4}\right)^4 \left(\frac{3}{4}\right)^6$
(ii) $1 - \left(\frac{3}{4}\right)^{10}$</td><td>3</td></tr>
<tr><td>10.</td><td>Mean: 4.6
Variance: 2.24
OR
Mean = 0.5</td><td>5</td></tr>
</table>

	Variance= $\frac{1}{8}$	
11.	**(i) k = 2** **(ii) F**(x) = $\begin{cases} 1 - e^{-2x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$ (iii) 0.	**5**

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