ONE TWO ACADEMY

Unit Test - 08

TOTAL:- 45 **GENERAL MATHEMATICS** **STD XII**

Differentials and Partial Derivatives

Choose the correct answer:- ***5 x 1 = 5***

1. If the percentage of the fifth root of 31 is approximately how many times the percentage error in 31?

(a) $\frac{1}{31}$ (b) $\frac{1}{5}$ (c) 5 (d) 31

2. $f(x,y,z) = xy + yz + zx$, then $f_x - f_z$ is equal to …..

(a) z-x (b) y-z (c) x-z (d) y-x

3. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$.

(a) $x + \frac{\pi}{2}$ (b) $-x + \frac{\pi}{2}$ (c) $x - \frac{\pi}{2}$ (d) $-x - \frac{\pi}{2}$

4. If $f(x) = \frac{x}{x+1}$ then its differential is given by

(a) $\frac{-1}{(1+x)^2}dx$ (b) $\frac{1}{(1+x)^2}dx$ (c) $\frac{1}{(1+x)}dx$ (d) $\frac{-1}{(1+x)}dx$

5. If $u(x,y) = x^2 + 3xy + y - 2019$, then $\frac{\partial u}{\partial x} \mid (4,-5)$ is equal to …….

(a) -4 (b) -3 (c) -7 (d) 13

Answer any 5 of the following (Question no 8 is compulsory):- ***5 x 2 = 10***

6)Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 3$ $dx = 0.02$.

7)Assume that the cross-section of the artery of a human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2mm to 2.1mm, how much is the cross-sectional area increased approximately?

8)Evaluate $\lim_{x \to (0.0)} \cos\left(\frac{e^x \sin y}{y}\right)$, if the limit exists.

9)The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of l.

10) Prove that $f(x,y) = x^3 - 2x^2y + 3xy^2 + y^3$ is homogenous.

11)For $g(x,y) = xe^y + 3x^2y$, find g_{xx} and g_{yy}.

12)If $f(x,y) = e^{xy}$, then find f_{xy}.

13) Verify Clariant's theorem for $f(x,y) = x^2+y^2$.

Answer any 5 of the following (Question no 21 is compulsory):- ***7 x 3 = 21***

14)Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

15)An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5mm and the radius to the outside of the shell is 5.3 mm, find the volume of the shell approximately.

16) If $U(x,y,z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$.

17)Find the linear approximation for the following functions at the indicated points.

$f(x) = x^3 - 5x + 12$, $x_0 = 2$.

18)The relation between the number of words y a person learns in x hours is given by $y = 52\sqrt{x}$, $0 \leq x \leq 9$. What is the approximate number of words learned when x changes from

(i) 1 to 1.1 hours.

(ii) 4 to 4.1 hours.

19) Find the partial derivatives of the following function at the indicated points.

$g(x,y) = 3x^2 + y^2 + 5x + 2$, (1,-2)

20) If $V(x,y) = e^x(x \cos y - y \sin y)$, then prove that $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$.

21)If $v(x,y) = \log(\frac{x^2+y^2}{x+y})$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$.

Answer the following:- **3 x 5 = 15**

22)Using the linear approximation to find approximate values of $(123)^{3/4}$.

[OR]

Assuming $\log_{10}e = 0.4343$, find an approximate value of $\log_{10}1003$.

23) If $v(x,y,z) = x^3 + y^3 + z^3 + 3xyz$, show that $\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$.

[OR]

If $w(x,y,z) = x^2 + y^2 + z^2$, $x = e^t$, $y = e^t \sin t$, and $z = e^t \cos t$, find $\frac{dw}{dt}$.

24) If $u(x,y) = \frac{x^2 + y^2}{\sqrt{x + y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$ a)Using Euler's theorem.

b)Without using Euler's theorem.

[OR]

Let $z(x,y) = x^3 - 3x^2y^3$, where $x = se^t$, $y = se^{-t}$, $s,t \in \mathbf{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

All the best