UNIT-4 WORK, ENERGY AND POWER

MY REVISION TIMELINE:-

SUMMARY:-

- When a force $\vec{F}$ acting on a body displace it by d$\vec{r}$, then the work done by the force is W = $\vec{F} \cdot d\vec{r}$ = F.drcosθ
- The work done by variables force is defined by $\int_i^f \vec{F} \cdot d\vec{r}$
- The kinetic energy can also be defined in terms of momentum which given by K.E = $\frac{p^2}{2m}$
- The gravitational potential energy at a height h is given by U=mgh. When the elongation or compression is x, the spring constant is k then the spring potential energy is given by u = $\frac{1}{2}kx^2$
- The work done by a conservative force around the closed path is zero and for a non-conservative force it is not true.
- In the vertical circular motion, the minimum speed required by the mass to complete the circle is $\sqrt{5gr}$. Where r is the radius of the circle.
- Power is defined as the rate of work done or energy delivered. It is equal to P = $\frac{W}{t} = \vec{F} \cdot \vec{v}$
- The coefficient of restitution = $\frac{velocity\ of\ separation}{velocity\ of\ approach}$

FORMULAS:-

- W = Fdrcosθ
- Work energy theorem: W = ΔKE
- Relation between momentum and kinetic energy, KE = $\frac{p^2}{2m}$
- u = mgh (Potential energy)
- Elastic potential energy, u = $\frac{1}{2}kx^2$
- $F_x = \frac{-du}{dx}$
- Power P = $\frac{dW}{dt}$
- Relation between power and velocity: P = $\vec{F} \cdot \vec{v}$
- Coefficient of restitution e = $\frac{v_2 - v_1}{u_2 - u_1}$

MULTIPLE CHOICE QUESTIONS:-

1. A uniform force of $(2\hat{\imath} + \hat{\jmath})$N acts on the particle of mass 1kg. The particles displaces from position $(3\hat{\jmath} + \hat{k})$m to $(5\hat{\imath} + 3\hat{\jmath})$m. The work done by the force on the particle is

c) 10J

Solution:

$\vec{F} = (2\hat{\imath} + \hat{\jmath})N$

$M = 1kg$

$\vec{s} = (5\hat{\imath} + 3\hat{\jmath})$ - $(3\hat{\jmath} + \hat{k})$ = $(5\hat{\imath} - \hat{k})$

$W = \vec{F} \cdot \vec{s}$

$W = (2\hat{\imath} + \hat{\jmath}) \cdot (5\hat{\imath} - \hat{k})$

W = 10J

2. A ball of mass 1kg and another mass 2kg are dropped from a tall building whose height is 80cm. After, a fall of 40cm each towards Earth, their respective kinetic energies will be in the ratio of

d) 1:2

Solution:

$M_1 = 1kg$

$M_2 = 2kg$

$h = 80cm$

$KE = \frac{1}{2}mv^2$

$v^2 = u^2 + 2gh$

$u = 0$

$v^2 = mgh$

$KE = mgh$

$k_1 = 1 \times 9.8 \times 40$

$k_2 = 2 \times 9.8 \times 40$

⇨ ***k_1:k_2 = 1:2***

3. A body of mass 1kg is thrown upwards with a velocity 20ms^{-1}. It momentarily comes to rest after attaining a height of 18m. How much energy is lost due to air friction?

a) 20J

Solution:

$M = 1kg$

$v = 20ms^{-1}$

$h = 18m$

$KE = \frac{1}{2}mv^2$

$KE = \frac{1}{2}(1)(20)^2$

$KE = 200J$

$PE = mgh$

$PE = 1 \times 10 \times 18$

$PE = 180J$

Energy lost due to friction = 200 – 180 = 20J

4. An engine pumps water continuously through a hose. Water leaves the hose with a velocity v and m is the mass per unit length of the water of the jet. What is the rate at which kinetic energy is imparted to the water?

a) $\frac{1}{2}mv^3$

Solution:

$Velocity = v$

$Length = l$

$Mass_1 = m$

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Mass$_2$ = m.x

Mass of the water flowing through the jet $= \frac{mx}{t}$ $(\therefore \frac{x}{t} = v)$

$= mv$

Rate at which kinetic energy $= \frac{1}{2}mv^2$

$= \frac{1}{2}(mv)v^2$

$\boxed{= \frac{1}{2}mv^3}$

5. A body of mass 4m is lying in xy-plane at rest. It suddenly explodes into three pieces. Two pieces each of mass m move perpendicular to each other with equal speed v. The total kinetic energy generated due to explosion is

b) $\frac{3}{2}mv^2$

Solution:

By conservation of linear momentum,

$2mv_1 = \sqrt{2}mv$

$v_1 = \frac{v}{\sqrt{2}}$

Total KE generated $= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)(v_1)^2$

$= mv^2 + mv_1^2$

$= mv^2 + m(\frac{v}{\sqrt{2}})^2$

$= mv^2 + \frac{1}{2}mv^2$

$\boxed{= \frac{3}{2}mv^2}$

6. The potential energy of a system increases, if work is done
a) by the system against a conservative force

7. What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop?

c) $\sqrt{5gr}$

Solution:

$T_A - mg = \frac{mv^2}{R}$

Energy at A $= \frac{1}{2}mv_o^2 \rightarrow 1$

Energy at C $= \frac{1}{2}mv_c^2 + mg(2R) \rightarrow 2$

$T_C + mg = \frac{mv^2}{R}$ *where* $T_C \geq 0$

$mg = \frac{mv^2}{R}$

$v = \sqrt{gR}$

From 1 and 2

$\frac{1}{2}mv_o^2 = \frac{1}{2}mv_c^2 + mg(2R)$

$\frac{1}{2}mv_o^2 = \frac{1}{2}m(gR + 4gR)$

$\boxed{v_o = \sqrt{5gR}}$

8. The work done by the conservative force for a closed path is
b) zero

9. If the linear momentum of the object is increased by 0.1% , then the kinetic energy increases by

b) 0.2%

Solution:

$KE = \frac{p^2}{2m}$

$KE\ \alpha\ p^2$

$KE_i = k$

$\Delta KE = \Delta k$

$p_i = p$

$\Delta p = p + \frac{0.1}{100}p = 1.001p$

$\frac{p^2}{\Delta p^2} = \frac{k}{\Delta k} \Rightarrow \frac{p^2}{1.002p^2} = \frac{k}{\Delta k}$

$\Delta k = 1.002k$

$KE = \frac{\Delta k - k}{k} \times 100$

$KE = \frac{1.002k - k}{k} \times 100$

$KE = 0.002 \times 100$

$\boxed{KE = 0.2\%}$

10. If the potential energy of the particle is $\alpha - \frac{\beta}{2}x^2$ then force experience by the particle is

c) F = - βx

Solution:

$F(x) = \frac{-du}{dx}$, *u is the potential energy*

$F(x) = \frac{d}{dx}(\alpha - \frac{\beta}{2}x^2)$

$F(x) = -\frac{\beta(2x)}{2}$

$\boxed{F(x) = -\beta x}$

11. A wind-powered energy generator converts wind energy into electric energy. Assume that the generator converts a fixed fraction of wind energy intercepted by its electric energy. For wind speed v, the electric power output will be proportional to

c) v^3

Solution:

Density of air = ρ

Area of the bladers = A

The mass of the air contained in volume v is

m = PAx where x is length

$k = \frac{1}{2}mv^2 = \frac{1}{2}PAxv^2$

$P = \frac{W}{t} = \frac{k}{\Delta t} = \frac{\frac{PAxv^2}{2}}{\frac{x}{v}}$

$P = \frac{1}{2}PAv^3$

$\boxed{\Rightarrow\ P\ \alpha\ v^3}$

12. Two equal mass m_1 and m_2 are moving along the same straight line which v_1 is 5ms^{-1} and v_2 is 9ms^{-1}. If the collision is elastic, then calculate the velocities after the collision of m_1 and m_2.

c) -9ms^{-1} and 5ms^{-1}

Solution:

Here the body have equal mass, $m_1 = m_2$

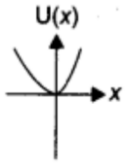
$v_1 = u_2$

$v_2 = u_1$

$$\boxed{v_1 = -9ms^{-1} \quad v_2 = 5ms^{-1}}$$

13. A particle is placed at the origin and a force F = kx is acting on it(When k is a positive constant) If u(0) = 0, the graph u(x) verse x will be (where u is the potential energy function)

c)



Solution:

$F(x) = \frac{-du}{dx}$

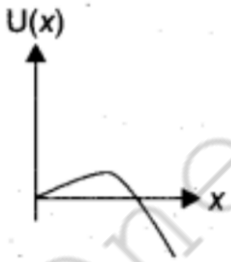
$$\int_0^{u(x)} du = -\int_0^x F\,dx = \int_0^x kx\,dx$$

$u(x) = \frac{kx^2}{2}$

$$\boxed{u(0) = 0}$$

14. A particle which is constrained to move along x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as F(x) = -kx + ax^3. Here k and a are positive constants. For x≥0, the functional form of the potential energy u(x) of the particle is

d)



Solution:

$d[u(x)] = -F\,dx$

$u(x) = \int_0^x -F\,dx$

$u(x) = \int_0^x (-kx + ax^3)\,dx$

$u(x) = \frac{kx^2}{2} - \frac{ax^4}{4}$

When x = 0, we get F = 0

At x = 0, $\frac{du}{dx} = 0$

Hence the slope of the graph is zero.

15. A spring of force constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of

b) $\frac{3}{2}k$

Solution:

l is the length of the string

k is a constant

Large piece of spring length = $\frac{2l}{3}$

Large piece of spring constant = x

$k \propto \frac{1}{l}$ →*1*

$x \propto \frac{1}{\frac{2l}{3}}$ →*2*

Divide equation 1 by 2

$\frac{k}{x} = \frac{2}{3}$

$\boxed{x = \frac{3}{2}k}$

NUMERICAL PROBLEMS:-

1. **Given:**

F = 30N

Load = 2kg

h = 10m

W = ?

Formula used:

W = F×r = mgh

Solution:

W = F×r

W = 30×10

$\boxed{W = 300J}$

2. **Given:**

$u = 5ms^{-1}$

Angle = 60°

e = 0.5

v = ?

Formula used:

The component of velocity parallel to the surface will remain unchanged.

$v\cos\alpha = u\cos\theta$

As per the law of restitution,

$v\sin\alpha = eu\sin\theta$

Solution:

$v\cos\alpha = u\cos\theta$

$v\cos\alpha = 5\cos 60°$

$v\cos\alpha = \frac{5}{2}$ →*1*

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$v\sin\alpha = (0.5)5\sin 60°$

$v\sin\alpha = \frac{1}{2} \times 5 \times \frac{\sqrt{3}}{2}$

$v\sin\alpha = \frac{5\sqrt{3}}{4} \rightarrow 2$

Squaring equation 1 and 2

$v^2\cos^2\alpha + v^2\sin^2\alpha = \frac{25}{4} + \frac{25\times3}{16}$

$v^2 = \frac{25\times4+25\times3}{16}$

$v^2 = 10.94$

$$\boxed{v = 3.31 ms^{-1}}$$

3. Given:

Velocity at lowest point $v_L = \sqrt{5gr}$

Velocity at highest point $v_T = \sqrt{gr}$

Solution:

Consider the triangle BED,

$\cos\theta = \frac{BE}{BD}$

$BE = r\cos\theta$

Now $EC = BC - BE$

$EC = r - r\cos\theta$

From law of conservation of energy

$\frac{1}{2}mv_T^2 + mgh = \frac{1}{2}mv_L^2$

$\frac{1}{2}mv_T^2 + mg(r - r\cos\theta) = \frac{1}{2}mv_L^2$

Divide by m on both sides

$\frac{1}{2}v_T^2 + g(r - r\cos\theta) = \frac{1}{2}v_L^2$

$v_T^2 - v_L^2 = -2g(r - r\cos\theta)$

$v_T^2 - 5gr = -2gr(1 - \cos\theta)$

If $\theta = 60°$

$v_T^2 - 5gr = -2gr(1 - \frac{1}{2})$

$v_T^2 - 5gr = -gr$

$v_1^2 = 4gr$

$$\boxed{v_1 = \sqrt{4gr}\ ms^{-1}}$$

4. Given:

$v_B^1 = \frac{v}{2}$

$v_B = v$

$\varphi = ?$

Formula used:

In x direction: $m_B v_B = 0 + m_A v_A^1 \cos\varphi \rightarrow 1$

In y direction: $0 = m_B v_B^1 - m_A v_A^1 \sin\varphi \rightarrow 2$

Divide equation 2 by 1

$$\frac{m_A v_A^1 \sin\varphi}{m_A v_A^1 \cos\varphi} = \frac{m_B v_B^1}{m_B v_B}$$

$$\tan\varphi = \frac{v_B^1}{v_B}$$

Solution:

$$\tan\varphi = \frac{v_B^1}{v_B}$$

$$\tan\varphi = \frac{\frac{v}{2}}{v} = \frac{1}{2}$$

$\varphi = 26.6°$

$\boldsymbol{\varphi = 26° 36´}$

5. Given:

Mass of the bullet $m_1 = 20g = 0.02\ kg$

Mass of the pendulum $m_2 = 5kg$

$h = 10cm = 0.1m$

Velocity of the bullet $= u_1 = ?$

Velocity of the pendulum $= u_2 = 0$ *(Pendulum is at rest)*

Final velocity $= v$

Formula used:

$m_1 u_1 + m_2 u_2 = (m_1 + m_2)v$

$v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$

From third equation of motion,

$v^2 - u^2 = 2gh$

$v^2 = 2gh$

Solution:

$v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$

$v = \frac{0.02 \times u_1 + 5 \times 0}{0.02 + 5}$

$v = \frac{0.02 u_1}{5.02} \rightarrow 1$

$v^2 = 2gh$

$v = \sqrt{2gh}$

$v = \sqrt{2 \times 9.8 \times 0.1}$

$v = 1.4\ ms^{-1} \rightarrow 2$

Substitute 2 in 1

$1.4 = \frac{0.02 u_1}{5.02}$

$\boldsymbol{u_1 = 351.4\ ms^{-1}}$

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