One two academy

Volume 2 Important Questions

CHAPTER 7

Two Marks

1. Evaluate $\lim_{x \to 0} \frac{xe^x - \sin x}{x}$.
2. A particle is fixed straight up from the ground to reach a height of s feet in t seconds when s = $128t - 16t^2$. Compute the maximum height of the particle reached.
3. For the function $f(x) = x^4 - 2x^2$, find all the values of c in (-2,2) such that f'(c) = 0.
4. Find the intervals of monotonicity for the function $f(x) = x^2 - 4x + 4$.
5. Evaluate: $\lim_{x \to 0} \left(\frac{\sin x}{x^2}\right)$.
6. Explain why Lagrange's MVT is not applicable for the function $f(x) = \left|\frac{1}{x}\right|$, $x \in [-1,1]$.

Three Marks

1. Find the points on the curve $y = x^3 - 6x^2 + x + 3$ where the normal is parallel to the line x + y = 1729.
2. Examine the concavity for the function $f(x) = x^4 - 4x^3$.
3. Show that the value in the conclusion of the MVT for $f(x) = Ax^2 + Bx + C$ on any interval [a,b].
4. Find the local extrema for the function $f(x) = x^2e^{-2x}$ using the second derivative test.
5. Find two positive numbers whose product is 20 and their sum is minimum.
6. Find the equation of the tangent to the curve $x^2y - x = y^3 - 8$ at x = 0.
7. Prove that the function f(x) = x - sin x is increasing but not strictly on the real line. Also, discuss the existence of local extrema.

Five Marks

1. A hollow cone with base radius a cm and height b cm is placed on a table. Show the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone
2. Expand log(1+x) as a Maclaurin's series up to 4 non-zero terms for $-1<x \leq 1$.
3. Find intervals of concavity and points of inflexion for the function $f(x) = \frac{1}{2}(e^x - e^{-x})$.
4. Find the absolute extrema of the function $f(x) = 3x^4 - 4x^3$ on the interval [-1,2].
5. Prove that among all the rectangles of the given area square has the least perimeter.
6. Find the angle between $y = x^2$ and $y = (x-3)^2$.
7. The volume of a cylinder equal V cubic cm, where V is a constant. Find the condition that minimises the total surface area of the cylinder.
8. If the curve $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$.
9. If we blow into a balloon of spherical shape at a rate of $1000cm^3$ per second. At what rate the radius of the balloon changes when the radius is 76 cm? Also, compute the rate at which the surface area changes.
10. Prove that $\lim_{n \to \infty} Ao\left(1 + \frac{r}{n}\right)^{nt} = A_oe^{rt}$

CHAPTER 8

Two Marks

1. $f(x,y) = x^3 - 3x^2 + y^2 + 5x + 6$, then find f_x at (1,-2).

Three Marks

1. If $u(x,y) = \dfrac{x^2 + y^2}{\sqrt{x+y}}$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{3}{2}u$.
2. If $U = \log(x^3 + y^3 + z^3)$, find $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z}$.
3. A circular plate uniformly under the influence of heat. If it's radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area and then the approximate percentage change in area.
4. If $u = e^{2(x-y)}$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = u \log u$.
5. Find the approximate value of $(31)^{\frac{1}{5}}$.
6. If $w(x,y) = xy + \sin(xy)$, then prove that $\dfrac{\partial^2 w}{\partial y \partial x} = \dfrac{\partial^2 w}{\partial x \partial y}$.

Five Marks

1. If $u = xyz$, $x = e^{-t}$, $y = e^{-t} \sin t$, $z = \sin t$ find $\dfrac{du}{dt}$.
2. If $f(x,y) = \log\sqrt{x^2 + y^2}$, show that $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$.
3. For the function $f(x,y) = \dfrac{3x}{y + \sin x}$, find f_x, f_y, and show that $f_{xy} = f_{yx}$.
4. Let $w(x,y,x) = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}}$, $(x,y,z) \neq (0,0,0)$. Show that $\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} + \dfrac{\partial^2 w}{\partial z^2} = 0$.
5. Let $z(x,y) = xe^y + ye^{-x}$, $x = e^{-t}$, $y = st^2$, $s,t \in R$. Find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$.
6. If $u = \sec^{-1}\left(\dfrac{x^3 - y^3}{x + y}\right)$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2 \cot u$.

CHAPTER 9

Two Marks

1. Evaluate: $\displaystyle\int_{-\log 2}^{\log 2} e^{-|x|}\, dx$.
2. Evaluate: $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \cos^2 x} dx$.
3. Evaluate: $\displaystyle\int_0^{\frac{\pi}{2}} \sin^{10} x\, dx$.
4. Evaluate: $\displaystyle\int_0^1 \frac{(\sin^{-1} x)^2}{\sqrt{1 - x^2}} dx$.
5. Find the area of the region bounded by the line $6x + 5y = 30$, x-axis and the lines $x = -1$ and $x=3$.

6. Evaluate: $\int_0^1 \frac{|x|}{x} dx.$

Three Marks

1. Evaluate: $\int_0^1 \frac{2x}{1+x^2} dx.$
2. Evaluate: $\int_0^{\frac{\pi}{2}} (sin^2 x \ + \ cos^4 x) dx$
3. Evaluate: $\int_0^{\pi} x^2 cos nx \ dx$, where n is a positive integer.
4. Evaluate $\int_0^{2\pi} x \ log\left(\frac{3 \ + \ cos \ x}{3 \ - \ cos \ x}\right) dx$ using the properties of integration.
5. Find the area of the region bounded by 2x - y +1 = 0, y = -1, y = 3 and y-axis.
6. Evaluate: $\int_0^{\frac{\pi}{3}} \frac{sec \ x \ tan \ x}{1 \ + sec^2 x} dx$

Five Marks

1. Find the area enclosed by the curve $y = -x^2$ and the straight line x + y + 2 = 0.
2. Evaluate: $\int_0^{2a} x^2 \sqrt{2ax - x^2} \ dx.$
3. Find the area of the region bounded between the parabola $x^2 = y$ and the curve y = |x|.
4. Using integration find the area of the region bounded by the triangle ABC, whose vertices A, B and C are (-1,1) , (3,2) and (0,5) respectively.
5. Using integration find the area of the region bounded by the triangle ABC, whose vertices A, B and C are (-1,2) , (1,5) and (3,4) respectively.
6. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{4 sin^2 x + 5 cos^2 x}.$
7. Prove that $\int_0^{\frac{\pi}{4}} log(1 + tan \ x) dx = \frac{\pi}{8} log 2.$
8. The curve $y = (x-2)^2 + 1$ has a minimum point at P. A point Q on the curve is such that the slope of PQ is 2. Find the area bounded by the curve and the chord PQ.

CHAPTER 10

Two Marks

1. Determine the order and degree (if exists) of the DE $(y'')^2 + (y')^2 = x \sin(y')$.
2. Find the differential equation of the curve represented by $xy = ae^x + be^{-x} + x^2$.
3. Show that the differential equation for the function $y = e^{-x} + mx + n$, where m and n are arbitrary constants is $e^x\left(\frac{d^2y}{dx^2}\right) - 1 = 0$.
4. Find the order and degree of the Differential equation $y \frac{dy}{dx} = \frac{x}{\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^3}$.
5. Solve: $\frac{dy}{dx} + y = e^{-x}.$
6. Form the differential equation of the family of the parabolas $y^2 = 4ax$, where a is an arbitrary constant.

Three Marks

1. Solve : cos x cos y dy - sin x sin y dx = 0.
2. Assume that the spherical rain drop evaporates at a rate proportional to its surface area. Form a differential equation involving the rate of change of radius if the raindrop.
3. Solve: $\frac{dy}{dx} = e^{x+y} + x^2e^{x^3+y}$.
4. Solve: $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.
5. Form the differential equation of $y = e^{3x}(C\cos 2x + D\sin 2x)$, where C and D are arbitrary constants.
6. Solve: $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$.
7. The population of a city grows at the rate of 5% per year. Calculate the time taken for the population to double. [log 2 = 0.6912].

Five Marks

1. The equation of electromotive force for an electric circuit containing resistance an self-inductance is $E = Ri + L\frac{di}{dt}$, where E is the electromotive force given to the circuit, R is the resistance and L, is the coefficient of induction. Find the current i at time t when E = 0.
2. Solve $y^2 - x^2\frac{dy}{dx} = xy\frac{dy}{dx}$.
3. Solve: $(1+2e^{x/y})\,dx + 2e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$.
4. Solve: $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$.
5. An equation relating to the stability of aircraft is given by $\frac{dv}{dt} = g\cos\alpha - kv$, where g , α, k are constants and v is the velocity. Obtain an expression interns of v if v = 0 when t = 0.
6. Solve: $\left(y - e^{\sin^{-1}x}\right)\frac{dx}{dy} + \sqrt{1-x^2} = 0$.

CHAPTER 11

Two Marks

1. Suppose a discrete random variable can only take 0, 1 and 2. The pmf is defined by $f(x) = \frac{x^2+1}{k}$, for x = 0,1,2. Find the value of k.
2. Find the mean of the distribution $f(x) = 3e^{-3x}$, $0 < x < \infty$.
3. If X is the random variable with distribution function F(x) given by,
$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}(x^2+x), & 0 \le x < 1 \\ 1, & x \ge 1 \end{cases}$$ then find the pdf f(x).
4. Find the mean and variance off X for the probability mass function of X given below:
$$f(x) = \begin{cases} 2(x-1), & 1 < x < 2 \\ 0 & otherwise \end{cases}$$
5. P(X=k) for the binomial distribution, B(n,p) = where n = 10 and $p = \frac{1}{5}$, k = 4.

Three Marks

1. If μ and σ^2 are the mean and variance of the discrete random variable X, and E(X+3) = 10 and $E(X+3)^2 = 116$, find μ and σ^2.
2. The probability distribution of a random variable is given below

X=x	0	1	2	3	4	5	6	7
P(X=x)	0	K	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Then find P(0<X<4).

3. The mean and variance of the binomial variate X are respectively 2 and 1.5. Find P(X=0).
4. If X ~ B(n,p) such that 4P(X=4) = P(X=2) and n = 6. Find the distribution, mean and standard deviation.

Five Marks

1. The probability density function of X is given by f(x) = $\begin{cases} ke^{\frac{-x}{3}} & for\ x > 0 \\ 0 & for x \leq 0 \end{cases}$

Find (i) the value of k (ii) P(X<3).

2. The commutative distribution function of a discrete random variable is given by

F(x) = $\begin{cases} 0 & -\infty < x < -1 \\ 0.15 & -1 \leq x < 0 \\ 0.35 & 0 \leq x < 1 \\ 0.60 & 1 \leq x < 2 \\ 0.85 & 2 \leq < x < 3 \\ 1 & 3 \leq x < \infty \end{cases}$, Find (i) the pmf (ii) P(X<1) and (iii) P(X $\geq$ 2)

3. The pdf of X is given by f(x) = f(x) = $\begin{cases} 16\ x\ e^{-4x} & for\ x > 0 \\ 0 & for x \leq 0 \end{cases}$ Find the mean and variance of X.
4. The sum of the mean and variance of a binomial distribution for five trials is 1.8. Find distribution.
5. Four fair coins are tossed once. Find the probability mass function, mean and variance for the number of heads that occurred
6. An MCQ exam has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (ii) atlas one correct answer.

CHAPTER 12

Two Marks

1. Verify the associative property under the binary operation * defined by $a*b = a^b$, ∀ a,b $\in N$.
2. Establish the equivalence property: p—>q ≡ - p v q.
3. On Z, define * by $(m*n) = m^n + n^m$: ∀ m,n $\in Z$. Is * binary on Z?
4. Let A = { a + $\sqrt{5}b$: a, b $\in Z$ }. Check whether the usual multiplication is a binary operation on A.
5. Write in the statements in words corresponding to -p , qv-p, where p is 'It is cold' and q is 'It is raining'
6. Let a*b = a + b + ab - 7. Is * binary on **R**?

Three Marks

1. Show that (i) p v (-p) is a tautology. (ii) p ^ (-p) is a contradiction.

Five Marks

1. Verify whether the compound proposition (p—>q) <—> (-p—>q) is a tautology, contradiction or contingency.
2. Using truth table check whether the statements -(pvq)v(-p^q) and - p are logically equivalent.
3. Verify the (i) closure property (ii) associative property (iii) existence of identity, (iv) existence f inverse and (v) commutative property for the operation $+_5$ on Z_5 using the table corresponding to addition modulo 5.
4. Establish the equivalence property p<—> q ≡ (p—>q)^(q—>p).
5. Prove that p—>(-qvr) ≡ - p v (-qvr) using truth table.
6. Show that p<—> q ≡ (p^q) V (-p ^ -q).