

Vector Algebra and 3D Geometry

Choose the correct answer:-

5 x 1 = 5

1.If $[\vec{a}, \vec{b}, \vec{c}] = 1$, then the value of $\frac{[\vec{a}, \vec{b}, \vec{c}]}{[\vec{c}, \vec{a}, \vec{b}]} + \frac{[\vec{b}, \vec{c}, \vec{a}]}{[\vec{a}, \vec{b}, \vec{c}]} + \frac{[\vec{c}, \vec{a}, \vec{b}]}{[\vec{c}, \vec{b}, \vec{a}]}$ is

- (1) 1 (2) -1 (3) 2 (4) 3

2.The volume of the parallelepiped with its edges represented by the vectors $\vec{i} + \vec{j}$, $\vec{i} + 2\vec{j}$ and $\vec{i} + \vec{j} + \pi\vec{k}$ is

- (1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{1}$ (4) $\frac{\pi}{4}$

3.If \vec{a} and \vec{b} are parallel vectors, then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to

- (1) 2 (2) -1 (3) 1 (4) 0

4.If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3 + \alpha z + \beta$, then (α, β) is

- (1) 5,(-5,5) (2) (-6,7) (3) (5,-5) (4) (6,-7)

5.Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

- (a) 0 (2) 1 (3) 2 (4) 3

Answer any 5 of the following (Question no 13 is compulsory):-

5 x 2 = 10

6)If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar vectors, show that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$

7)Show that the points (2,3,4), (-1,4,5) and (8,1,2) are collinear.

8)Find the intercepts cut off by the plane $\vec{r} \cdot (6\vec{i} + 4\vec{j} - 3\vec{k}) = 12$ on coordinate axes.

9)Verify whether the line $\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{12}$ lies in the plane $5x - y + z = 8$.

10) A particle acted on by constant forces $8\vec{i} + 2\vec{j} - 6\vec{k}$ and $6\vec{i} + 2\vec{j} - 2\vec{k}$ is displaced from the point (1,2,3) to point (5,4,1). Find the total work done by the forces.

11) Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $-6\vec{i} + 14\vec{j} + 10\vec{k}$, $14\vec{i} - 10\vec{j} - 6\vec{k}$ and $2\vec{i} + 4\vec{j} - 2\vec{k}$.

12) If \vec{a} , \vec{b} , \vec{c} and \vec{d} are coplanar vectors then show that $((\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})) = \vec{0}$.

13) Find a parametric form of vector equation of plane which is at distance of 7 units from the origin having 3, -4, 5 as direction ratios of a normal to it.

Answer any 5 of the following (Question no 21 is compulsory):-

7 x 3 = 21

14) a) What is a zero vector ? b) What is a coterminous vector ? c) State Apollonius theorem.

15) Prove by vector method that an angle in a semi circle is right angle.

16) Show that the four points (6,-7,0), (16,-19,-4), (0,3,-6) and (2,-5,10) lie on a same plane.

17) Show that the straight lines $x+1 = 2y = -12z$ and $x = y+2 = 6z - 6$ are skew and hence find the shortest distance between them.

18) If the straight lines $\frac{x-1}{2} = \frac{y+1}{\alpha} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{\alpha}$ are coplanar, find α .

19) Find the distance of the points (5,-5,-10) from the point of intersection of a straight line passing through the points A(4,1,2) and B(7,5,4) with the plane $x - y + z = 5$.

20) For any vector \vec{a} , prove that $\vec{i} \times (\vec{a} \times \vec{i}) + \vec{j} \times (\vec{a} \times \vec{j}) + \vec{k} \times (\vec{a} \times \vec{k}) = 2\vec{a}$.

21) Find the angle between the line $\vec{r} = (2\vec{i} - \vec{j} + \vec{k}) + t(\vec{i} + 2\vec{j} - 2\vec{k})$ and the plane $\vec{r} \cdot (6\vec{i} + 3\vec{j} + 2\vec{k}) = 8$.

Answer the following:-

3 x 5 = 15

22) Prove by vector method $\cos(A-B) = \cos A \cos B + \sin A \sin B$.

[OR]

Prove by vector method that the perpendiculars from the vertices to the opposite sides of a triangle are concurrent.

23) If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = -1\vec{i} + 2\vec{j} - 4\vec{k}$, $\vec{c} = \vec{i} + \vec{j} + \vec{k}$ then prove that

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}.$$

[OR]

Find the foot of the perpendicular drawn from the point (5,4,2) to the line

$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}. \text{ Also, find the equation of the perpendicular.}$$

24) Find a non-parametric form of vector equation and cartesian equation of the plane passing through the point (1,-2,4) and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line

$$\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}.$$

[OR]

If the vectors $a\vec{i} + a\vec{j} + c\vec{k}$, $\vec{i} + \vec{k}$ and $c\vec{i} + c\vec{j} + b\vec{k}$ are coplanar, prove that c is the GM of a and b.

All the best