ONE TWO ACADEMY WEEKLY EXAM - 1

Total:- 50marks | **MATHEMATICS** | **HSC 1ST YEAR**

Choose the correct answer:- **10 x 1 = 10**

1) If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is Cofactors of a_{ij}, then value of Δ is given by

(A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ (B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$

(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$ (D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

2) If A is skew-symmetric of order n and C is a column matrix of order $n \times 1$, then $C^T AC$ is

(1) an identity matrix of order n (2) an identity matrix of order 1

(3) a zero matrix of order 1 (4) an identity matrix of order 2

3) If the square of the matrix $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is the unit matrix of order 2, then α, β and γ should satisfy the relation.

(1) $1+\alpha^2+\beta\gamma=0$ (2) $1-\alpha^2-\beta\gamma=0$

(3) $1-\alpha^2+\beta\gamma=0$ (4) $1+\alpha^2-\beta\gamma=0$

4) If A is a square matrix, then which of the following is not symmetric?

(1) $A+A^T$ (2) AA^T (3) A^TA (4) $A-A^T$

5) If A and B are symmetric matrices of order n, where $(A \neq B)$, then

(1) $A+B$ is skew-symmetric (2) $A+B$ is symmetric

(3) $A+B$ is a diagonal matrix (4) $A+B$ is a zero matrix

6) If $A = \begin{bmatrix} a & x \\ y & a \end{bmatrix}$ and if $xy=1$, then det $(A\,A^T)$ is equal to

(1) $(a-1)^2$ (2) $(a^2+1)^2$ (3) a^2-1 (4) $(a^2-1)^2$

7) If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, and $A + A' = I$, then the value of α is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) π (D) $\frac{3\pi}{2}$

8) If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

(A) 6 (B) ± 6 (C) -6 (D) 0

9) Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \le \theta \le 2\pi$. Then

(A) Det (A) = 0 (B) Det (A) $\in$ (2, ∞)

(C) Det (A) $\in$ (2, 4) (D) Det (A) $\in$ [2, 4]

10) If a, b, c, are in A.P, then the determinant

$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$ is

(A) 0 (B) 1 (C) x (D) $2x$

Answer the following:- **5 x 2 = 10**

1) Find the sum $A + B + C$ if A, B, C are given by

$$A = \begin{bmatrix} \sin^2\theta & 1 \\ \cot^2\theta & 0 \end{bmatrix},\ B = \begin{bmatrix} \cos^2\theta & 0 \\ -\text{cosec}^2\theta & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}.$$

2) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, show that A^2 is a unit matrix.

3) Let A and B be two symmetric matrices. Prove that $AB = BA$ if and only if AB is a symmetric matrix.

4) Find the value of x if $\begin{vmatrix} x-1 & x & x-2 \\ 0 & x-2 & x-3 \\ 0 & 0 & x-3 \end{vmatrix} = 0$.

5) Compute $|A|$ using Sarrus rule if $A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}$

Answer the following:- **5 x 3 = 10**

6) For any square matrix A with real number entries, $A + A^T$ is a symmetric matrix and $A - A^T$ is a skew-symmetric matrix.

7) If each element in a row (or column) of a matrix is multiplied by a scalar k, then the determinant is multiplied by the same scalar k.

8) If elements of a row (or column) are multiplied with corresponding cofactors of any other row (or column) then their sum is zero.

9) Find the equation of the line joining A(1, 3) and B (0, 0) using determinants and find k if D(k, 0) is a point such that area of triangle ABD is 3sq units.

10) Prove that $\begin{vmatrix} 1 & x & x \\ x & 1 & x \\ x & x & 1 \end{vmatrix}^2 = \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}$

Answer the following:- **3 x 5 = 15**

11) If A_i, B_i, C_i are the cofactors of a_i, b_i, c_i, respectively, $i = 1$ to 3 in $|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, show that $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = |A|^2$.

12) Prove that $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$.

13) Show that $\begin{vmatrix} a^2+x^2 & ab & ac \\ ab & b^2+x^2 & bc \\ ac & bc & c^2+x^2 \end{vmatrix}$ is divisible by x^4.