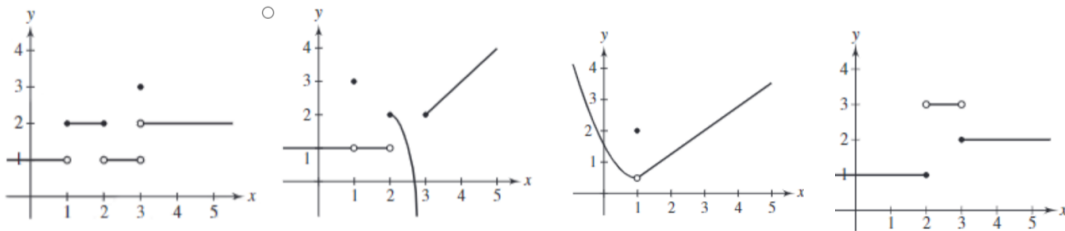


**Answer any three of the following:- 3x5=15**

- 1) Evaluate :  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$ .
- 2) Evaluate the following limit  $\lim_{x \rightarrow 0} \frac{\sin x(1 - \cos x)}{x^3}$
- 3) **Prove that**  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$
- 4) Show that  $\lim_{x \rightarrow 0^+} x \left[ \left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{2}{x} \right\rfloor + \cdots + \left\lfloor \frac{15}{x} \right\rfloor \right] = 120$ .
- 5) **Prove that**  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$ .  
and hence evaluate  $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$

- 1  $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{x}$   
(1) 0 (2) 1 (3)  $\sqrt{2}$  (4) does not exist
- 2  $\lim_{\theta \rightarrow 0} \frac{\sin \sqrt{\theta}}{\sqrt{\sin \theta}}$   
(1) 1 (2) -1 (3) 0 (4) 2
- 3  $\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{x} =$   
(1) 1 (2)  $e$  (3)  $\frac{1}{e}$  (4) 0
- 4  $\lim_{x \rightarrow 0} \frac{e^{\tan x} - e^x}{\tan x - x} =$   
(1) 1 (2)  $e$  (3)  $\frac{1}{2}$  (4) 0
- 5 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = \begin{cases} x & x \text{ is irrational} \\ 1-x & x \text{ is rational} \end{cases}$  then  $f$  is  
(1) discontinuous at  $x = \frac{1}{2}$  (2) continuous at  $x = \frac{1}{2}$   
(3) continuous everywhere (4) discontinuous everywhere
- 6 Assume the function  $f$  is left- continuous but not continuous at  $x = 2$  and right continuous at  $x = 3$ . Identify the graph of  $f$



7 The value of  $\lim_{x \rightarrow k^-} x - \lfloor x \rfloor$ , where  $k$  is an integer is

- (1) -1 (2) 1 (3) 0 (4) 2

8 At  $x = \frac{3}{2}$  the function  $f(x) = \frac{|2x-3|}{2x-3}$  is

- (1) continuous (2) discontinuous (3) differentiable (4) non-zero

9  $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} =$

- (1)  $\log ab$  (2)  $\log \left( \frac{a}{b} \right)$  (3)  $\log \left( \frac{b}{a} \right)$  (4)  $\frac{a}{b}$

10  $\lim_{x \rightarrow 0} \frac{8^x - 4^x - 2^x + 1^x}{x^2} =$

- (1)  $2 \log 2$  (2)  $2(\log 2)^2$  (3)  $\log 2$  (4)  $3 \log 2$

11 If  $f(x) = x(-1)^{\lfloor \frac{1}{x} \rfloor}$ ,  $x \leq 0$ , then the value of  $\lim_{x \rightarrow 0} f(x)$  is equal to

- (1) -1 (2) 0 (3) 2 (4) 4

**Answer any five of the following:- 5 x4 = 20**

1  $\lim_{x \rightarrow 3} \frac{x^2 - 81}{\sqrt{x} - 3}$

2  $\lim_{x \rightarrow 0} \frac{(2+x)^5 - 2^5}{x}$

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3  $\lim_{x \rightarrow \infty} \frac{x^3 + x}{x^4 - 3x^2 + 1}$

4  $\lim_{x \rightarrow 0} \frac{\tan 2x}{x}$

5  $\lim_{x \rightarrow 0} \frac{2^x - 3^x}{x}$

6  $\lim_{x \rightarrow 0} \frac{\sin(a+x) - \sin(a-x)}{x}$

7  $\lim_{x \rightarrow 0} \frac{\sqrt{x^2 + a^2} - a}{\sqrt{x^2 + b^2} - b}$

8  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

**Answer any five of the following:- 5 x4 = 20**

1 Find the constant  $b$  that makes  $g$  continuous on  $(-\infty, \infty)$ .

2  $g(x) = \begin{cases} x^2 - b^2 & \text{if } x < 4 \\ bx + 20 & \text{if } x \geq 4 \end{cases}$

3 If  $f$  and  $g$  are continuous functions with  $f(3) = 5$

and  $\lim [2f(x) - g(x)] = 4$ , find  $g(3)$

4 For what value of  $\alpha$  is this function  $f(x) = \begin{cases} \frac{x^4 - 1}{x - 1}, & \text{if } x \neq 1 \\ \alpha, & \text{if } x = 1 \end{cases}$  continuous at  $x = 1$ ?

5 Evaluate :  $\lim_{x \rightarrow \infty} \left( \frac{x+2}{x-2} \right)^x$ .

6 Prove that  $\lim_{x \rightarrow 0} \sin x = 0$ .

7 Calculate  $\lim_{x \rightarrow -1} (x^2 - 3)^{10}$ .

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