One Two academy | Matrices and Determinants

Maximum Mark: 25 **Mathematics** **Time: 50 minutes**

Choose the correct answer: **3 x 1 = 3**

1. If A^TA^{-1} is symmetric, then A^2 =

(a) A^{-1} (b) $(A^T)^2$ (c) A^T (d) $(A^T)^{-1}$

2. If $|adj(adjA)| = |A|^9$, then the order of the square matrix A is

(a) 3 (b) 4 (c) 2 (d) 5

3. The rank of the of the diagonal matrix

$$\begin{bmatrix} -1 & & & & \\ & 2 & & & \\ & & 0 & & \\ & & & -4 & \\ & & & & 0 \end{bmatrix}$$

(a) 0 (b) 2 (c) 3 (d) 5

Answer the following: **3 x 2 = 6**

4. Solve the following systems of linear equations by Cramer's rule:

$$\frac{3}{x} + 2y = 12$$

$$\frac{2}{x} + 3y = 13$$

5. If A is a non-singular matrix of odd order, prove that | adj A | is positive.

6. Find the inverse of the matrix $\begin{pmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{pmatrix}$.

Answer any two of the following: **2 x 3 = 6**

7. A bag contains 3 types of coins namely Re. 1, Rs. 2 and Rs. 5. There are 30 coins amounting to Rs. 100 in total. Find the number of coins in each category.

8. Find the condition on a b, and c so that the following system of linear equations has one parameter family of solutions:

$$x + y + z = a,$$

$$x + 2y + 3z = b,$$

$$3x + 5y + 7z = c.$$

9. Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

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Answer the following: **2 x 5 = 10**

10.Find the value of k for which the equations

$$kx - 2y + z = 1,$$

$$x - 2ky + z = -2,$$

$$x - 2y + kz = 1$$

have (i) no solution (ii) unique solution (iii) infinitely many solution.

11. If A = $\begin{bmatrix} 1 & \tan x \\ -tanx & 1 \end{bmatrix}$, Show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

OR

By using the Gaussian elimination method, balance the chemical reaction equation:

$$C_5H_8 + O_2 \rightarrow CO_2 + H_2O.$$

All the Best

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Answer Key attached in the next page !!!

This page is to prevent seeing answer key

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Answer Key

Q.no	Solution	Marks
1.	(b) $(A^T)^2$	**1**
2.	(b) 4	**1**
3.	(c) 3	**1**
4.	x = ½ , y = 3	**2**
5.	**Correct proof**	**2**
6.	$A^{-1} = \frac{1}{\lvert A\rvert}(\text{adj } A) = \frac{1}{(-1)}\begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$.	**2**
7.	Let x, y and z be the number of coins respectively in each category Re. 1, Rs. 2 and Rs. 5. From the given information $x + y + z = 30$ (i) $x + 2y + 5z = 100$ (ii) Here we have 3 unknowns but 2 equations. We assign arbitrary value k to z and solve for x and y. (i) and (ii) become $x + y = 30 - k$ $x + 2y = 100 - 5k \quad k \in R$ $\Delta = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1,\ \Delta_x = \begin{vmatrix} 30-k & 1 \\ 100-5k & 2 \end{vmatrix} = 3k - 40,\ \Delta_y = \begin{vmatrix} 1 & 30-k \\ 1 & 100-5k \end{vmatrix} = 70 - 4k$ By Cramer's Rule $x = \frac{\Delta_x}{\Delta} = 3k - 40, \quad y = \frac{\Delta_y}{\Delta} = 70 - 4k$ The solution is $(x, y, z) = (3k - 40, 70 - 4k, k) \quad k \in R$. Since the number of coins is a non-negative integer, $k = 0, 1, 2 \ldots$ Morever $3k - 40 \geq 0$, and $70 - 4k \geq 0 \Rightarrow 14 \leq k \leq 17$ $\therefore$ The possible solutions are (2, 14, 14), (5, 10, 15), (8, 6, 16) and (11, 2, 17) ***Note: One can use Gaussian elimination method.***	**3**
8.	c=a+2b	**3**
9.	18 days, 36 days	
10.	Case (i) : When k ≠ 1, the system has a unique solution Case (ii) : When k = 1, the system reduces to a single equation x + y + z = 1. The system can have more than one solution. Case (iii) : When k = -2, the system is inconsistent $\therefore$ ρ(A, B) ≠ ρ(A); $\therefore$ the system has no solution.	**5**
11.	Correct proof **OR** $C_5H_8 + 7O_2 \rightarrow 5CO_2 + 4H_2O.$	**5**