UNIT-11 WAVES

MY REVISION TIMELINE:-

SUMMARY:-

- A distance which carries energy and momentum from one point in space to another point in space without the transfer of medium is known as wave.
- The wave which require medium for their propagation is called mechanical waves.
- The wave which do not require medium for their propagation is called non-mechanical waves.
- For a transwaves wave, the vibration of particles in a medium in perpendicular to the direction of propagation of the wave.
- For a longitudinal wave, the vibration of particles in a medium in parallel to the direction of propagation of the wave.
- Elasticity and inertia are necessary properties of the medium of wave propagation.
- Waves formed in still water (ripples) are transverse and wave formed due to vibration of tuning fork is longitudinal.
- The distance between two consecutive crests or troughs is known as wavelength, λ.
- The number of waves which crossed a point per second is known as frequency, f.
- The time taken by one wave to cross a point is known as time period, T.
- Velocity of the wave is v = λf
- Frequency is source dependent and velocity is medium dependent.
- The velocity of transverse wave produced in a stretched string depends on tension in the string and mass per unit length. It does not depend on shape of the wave form.
- Velocity of transverse wave on a string is v = $\sqrt{\frac{T}{\mu}} ms^{-1}$
- Velocity of longitudinal wave in an elastic medium is v = $\sqrt{\frac{E}{\rho}} ms^{-1}$
- The minimum distance from a sound reflecting wall to hear an echo at 20°C is 17.2 metres.
- The wave equation is $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$ in one space dimension.
- Wave number is given by k = $\frac{2\pi}{\lambda}\ radm^{-1}$
- During interference, the resultant intensity is $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$, where the intensity is square of the amplitude $I = A^2$.

  For constructive interference, $I_{maximum} = (\sqrt{I_1} + \sqrt{I_2})^2 = (A_1 + A_2)^2$

  For destructive interference, $I_{minimum} = (\sqrt{I_1} - \sqrt{I_2})^2 = (A_1 - A_2)^2$

- When we superimpose two or more waves with slightly different frequencies then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called beat frequency.
- If natural frequencies are written as integral multiples of fundamental frequencies then the frequencies are said to be harmonics. Thus the first harmonics is called the fundamental frequency, the second harmonics is $v_2 = 2v_1$, the third harmonics is $v_3 = 3v_1$, and so on.
- Loudness of sound is defined as "the degree of sensation of sound produced in the ear or the perception of sound by the listener".
- The intensity of sound is defined as "the sound power transmitted per unit area placed normal to the program of sound wave".
- Sound intensity level, $\Delta L = 10\log_{10}\frac{I_1}{I_0}$ decibel.
- In closed organ pipe, the frequencies of harmonic are in the ratio $f_1:f_2:f_3:f_4 = 1:3:5:7$
- A closed organ pipe has only odd harmonics and the corresponding frequency of the n^{th} harmonic is $f_n = (2n+1)f_1$.
- In open organ pipe, the frequencies of harmonic are in the ratio $f_1:f_2:f_3:f_4 = 1:2:3:4$
- An open organ pipe has all harmonics and frequency of the n^{th} harmonic is $f_n = nf_1$.
- When the source and the observer are in relative motion with respect to the medium in which sound propagates, the frequency of the sound wave observed is different from the frequency of the source. This phenomenon is called Doppler effect. The frequency received by the observer is known as apparent frequency.
- When the source moves towards a stationary observer, the apparent frequency $f' = f(1+\frac{v_s}{v})$
- When the source moves away from a stationary observer, the apparent frequency $f' = f\ \ (1-\frac{v_s}{v})$
- When the observer moves towards a stationary observer, $f' = f(1-\frac{v_s}{v})$
- When the observer recedes away from a stationary source, $f' = f(1+\frac{v_o}{v})$
- When the observer recedes away from a stationary source, $f' = f(1=\frac{v_o}{v})$
- When the source observer recedes away from a stationary source, $f' = f(1-\frac{v_o}{v})$
- When the source and the observer approach each other, $f' = f(\frac{v+v_o}{v-v_o})$
- When the source and the observer recedes from each other, $f' = f(\frac{v-v_o}{v+v_1})$
- When the source chases the observer, $f' = f(\frac{v-v_o}{v-v_s})$
- When the observer chases the source, $f' = f(\frac{v+v_o}{v+v_s})$

FORMULAS:-

- Velocity of transverse wave on a string is $v = \sqrt{\frac{T}{\mu}}ms^{-1}$
- Velocity of a longitudinal wave in an elastic medium is $v = \sqrt{\frac{E}{\rho}}ms^{-1}$

- The wave equation is $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$ in one space dimension.
- Wave number is given by k = $\frac{2\pi}{\lambda}\ radm^{-1}$
- The apparent frequency f´= f(1+$\frac{v_s}{v}$)
- When the source moves away from a stationary observer, f´= f(1- $\frac{v_s}{v}$)
- When the observer moves towards a stationary source, f´= f(1+ $\frac{v_o}{v}$)
- When the observer recedes away from a stationary source, f´= f(1- $\frac{v_o}{v}$)
- When the source and the observer approach each other, f´= f($\frac{v+v_o}{v-v_o}$)
- When the source and the observer recedes from each other, f´= f($\frac{v-v_o}{v+v_1}$)
- When the source chases the observer, f´= f($\frac{v-v_o}{v-v_s}$)
- When the observer chases the source, f´= f($\frac{v+v_o}{v+v_s}$)

MULTIPLE CHOICE QUESTIONS:-

1. A student tunes his guitar by striking a 120 Hertz with a tuning fork, and simultaneously plays the 4th string on his guitar. By keen observation, he hears the amplitude of the combined sound oscillating thrice per second. Which of the following frequencies is the most likely the frequency of the 4th string on his guitar?
b) 117
Solution:
F = 120 Hertz
Frequency of the 4th string is obtained from harmonic series
120, 119, 118 and 117 Hz

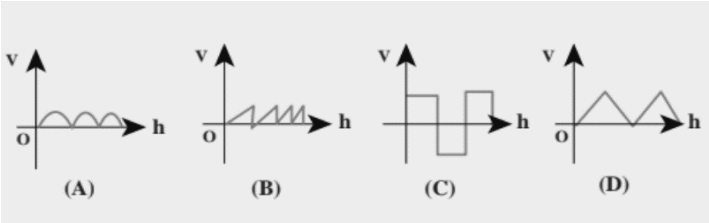
$\therefore f_4 = 117\ Hz$

2. A transverse wave moves from a medium A to a medium B. In medium A, the velocity of the transverse wave is 500 ms^{-1} and the wavelength is 5 m. The frequency and the wavelength of the wave in medium B when its velocity is 600 ms^{-1}, respectively are
d) 100Hz and 6m
Solution:
$v_A = 500\ ms^{-1}$ $\lambda_A = 5m$
Frequency in medium B
$f_B = \frac{v_A}{\lambda_A} = \frac{500}{5} = \mathbf{100Hz}$
Wavelength in medium B
$\lambda_B = \frac{v_S}{f_S} = \frac{600}{100} = \mathbf{6Hz}$

3. **For a particular tube, among six harmonic frequencies below 1000 Hz, only four harmonic frequencies are given: 300 Hz, 600 Hz, 750 Hz and 900 Hz. What are the two other frequencies missing from this list?**
   b) 150Hz, 450Hz
4. **Which of the following options is correct?**

A	B
(1) Quality	(A) Intensity
(2) Pitch	(B) Waveform
(3) Loudness	(C) Frequency

   a) (B), (C) and (A)
5. **Compare the velocities of the wave forms given below, and choose the correct option.**



   where, v_A, v_B, v_C and v_D are velocities given in (A), (B), (C) and (D), respectively.
   c) $v_A = v_B = v_C = v_D$
   Solution:
   The amplitude of the velocity wave is same in all four cases.

6. **A sound wave whose frequency is 5000 Hz travels in air and then hits the water surface. The ratio of its wavelengths in water and air is**
   a) 4.30
   Solution:
   Frequency = 5000Hz
   Speed of sound in air = 332 m/s
   Speed of sound in water = 1450 m/s
   Wavelength of sound in air $= \frac{332}{5000}$
   Wavelength of sound in air $= 66.4 \times 10^{-3}$
   Wavelength of sound in water $= \frac{1450}{5000}$
   Wavelength of sound in water $= 290 \times 10^{-3}$
   $\frac{\lambda_{water}}{\lambda_{air}} = \frac{290\times10^{3}}{66.4\times10^{-3}}$

   $\boxed{\frac{\lambda_{water}}{\lambda_{air}} = 4.367}$

7. **A person standing between two parallel hills fires a gun and hears the first echo after t_1 sec and the second echo after t_2 sec. The distance between the two hills is**
   d) $\frac{v(t_1+t_2)}{2}$

Solution:

$2d_1 = vt_1$ – first echo

$2d_2 = vt_2$ – second echo

$\therefore d = d_1 + d_2$

$$\therefore \frac{v(t_1+t_2)}{2}$$

8. An air column in a pipe which is closed at one end, will be in resonance with the vibrating body of frequency 83 Hz. Then the length of the air column is

c) 1.0m

Solution:

$l = \lambda$

$f = 8.3Hz$

$v = 332$ m/s

$\lambda = 4l$

$\lambda = \frac{v}{f} = \frac{332}{83} = 4m$

$l = \frac{\lambda}{4} = \frac{4lm}{4}$

$l = 1.0m$

9. The displacement y of a wave travelling in the x direction is given by $y = (2 \times 10^{-3}) \sin(300t - 2x + \frac{\pi}{4})$ where x and y are measured in metres and t in second. The speed of the wave is

a) 150 m/s

Solution:

The given equation is similar to $y = A \sin(\omega t - \frac{x}{\lambda})$

$\omega = 300$, $f = \frac{300}{2\pi} = \frac{150}{\pi}$

Speed $v = f\ \lambda$

Speed $v = \frac{150}{\pi} \times \pi$

Speed v = 150 m/s

10. Consider two uniform wires vibrating simultaneously in their fundamental notes. The tensions, densities, lengths and diameter of the two wires are in the ratio 8:1, 1:2, x:y and 4:1 respectively. If the note of the higher pitch has a frequency of 360 Hz and the number of beats produced per second is 10, then the value of x:y is

a) 36:35

Solution:

No. of beats = 10

Frequency of pitch in the first wire $f_1 = 360Hz$

Frequency of pitch in the second wire $f_2 = 360\text{-}10 = 350Hz$

Ratio of length = x:y

$l \alpha f$

$x:y = f_1:f_2$

$x:y = 360:350$

$\boxed{x:y = 36:35}$

11. Which of the following represents a wave?

d) sin(x+vt)

12. A man sitting on a swing which is moving to an angle of 60° from the vertical is blowing a whistle which has a frequency of 2.0 kHz. The whistle is 2.0 m from the fixed support point of the swing. A sound detector which detects the whistle sound is kept in front of the swing. The maximum frequency the sound detector detected is

a) 2.027 kHz

Solution:

$v = 2kHz$

Velocity of whistle (source) $v_s = 2m/s$

Velocity of sound $v = 332\ m/s$

Maximum frequency $v_{max} = \left[\frac{v+v_s}{v}\right] u$

$v_{max} = \left[\frac{332+2}{332}\right] 2$

$v_{max} = \left[\frac{334}{332}\right] 2$

$v_{max} = 1.006 \times 2$

$\boxed{v_{max} = 2.012\ kHz}$

13. Let y = $\frac{1}{1+x^2}$ at t = 0s be the amplitude of the wave propagating in the positive x-direction. At t = 2s, the amplitude of the wave propagating becomes y = $\frac{1}{1+(x-2)^2}$. Assume that the shape of the wave does not change during propagation. The velocity of the wave is

b) 1.0 ms^{-1}

Solution:

Factual information

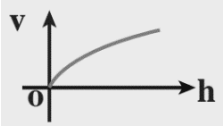
At t = 0, amplitude = $\frac{1}{1+x^2}$

At t = 2, amplitude = $\frac{1}{1+(x-2)^2}$

$\therefore v = \frac{\Delta y}{\Delta t} = \frac{2}{2}$

$\boxed{v = 1.0\ ms^{-1}}$

14. A uniform rope having mass m hangs vertically from a rigid support. A transverse wave pulse is produced at the lower end. Which of the following plots shows the correct variation of speed v with height h from the lower end?

d) 

15. An organ pipe A closed at one end is allowed to vibrate in its first harmonic and another pipe B open at both ends is allowed to vibrate in its third harmonic. Both A and B are in resonance with a given tuning fork. The ratio of the length of A and B is

c) $\frac{1}{6}$

Solution:

First harmonic of a closed organ pipe $L_e = \frac{3\lambda}{2}$

Third harmonic of an open organ pipe $L_o = \frac{3\lambda}{2}$

$\frac{L_e}{L_o} = \frac{\lambda}{4} \times \frac{2}{3\lambda}$

$$\boxed{\frac{L_e}{L_o} = \frac{1}{6}}$$

NUMERICAL PROBLEMS:-

1. Given:

Speed v = 900 m/s

In 120s, number of waves produced = 3000

Formula used:

Wavelength $\lambda = \frac{v}{f}$

Solution:

In 1s, number of waves produced $f = \frac{3000}{120} = 25Hz$

Wavelength $\lambda = \frac{v}{f}$

$\lambda = \frac{900}{25}$

$$\boxed{\lambda = 36m}$$

2. Given:

Number of molecules of Helium = 2

Number of molecules of Oxygen = 4

T = 300K

Formula used:

$v = \sqrt{\frac{rRT}{M}}$

Solution:

When helium and oxygen are mixed, the molecular weight of the mixture of gases is

$M_{mix} = \frac{n_1M_1+n_2M_2}{n_1+n_2}$

$M_{mix} = \left[\frac{2\times4+4\times32}{2+4}\right] 10^{-3} kg/mol$

$M_{mix} = \left[\frac{136}{6}\right] 10^{-3} kg/mol$

$M_{mix} = 22.6 \times 10^{-3} kg/mol$

In addition, Helium is monoatomic $C_{v1} = \frac{3R}{2}$ *and Oxygen is diatomic* $C_{v3} = \frac{5R}{2}$

For mixture $(Cv)_{mix} = \frac{n_1 C_{v1} + n_3 C_{v3}}{n_1 + n_3}$

$(Cv)_{mix} = \frac{2\times\frac{3}{2}R + 4\times\frac{5}{2}R}{2+4}$

$(Cv)_{mix} = \frac{13R}{6}$

$(Cp)_{mix} = (Cv)_{mix} + R$

$(Cp)_{mix} = \frac{13R}{6} + R = \frac{19R}{6}$

$\gamma_{mix} = \frac{C_p}{C_v} = \frac{\frac{19R}{6}}{\frac{13R}{6}} = \frac{19}{13}$

According to laplace, the velocity of sound is

$v = \sqrt{\frac{rRT}{M}}$

$v = \sqrt{\frac{19\times8.31\times300\times6}{13\times136\times10^{-3}}}$

$v = \sqrt{\frac{28420}{1768}} \times 10^4$

$$\boxed{v = 400.9\ ms^{-1}}$$

3. Solution:

In the first case:

Time = 3.5s

Velocity of sound wave is sea v = 1450 m/s

$2d_1 = vt$

$2d_1 = 1450 \times 3.5$

$2d_1 = 5075\ m$

Depth of the sea $d_1 = \frac{5075}{2} = 2537.5m$

In the second case:

Time = 2s

Velocity of sound wave is sea v = 1450 m/s

$2d_2 = vt$

$2d_2 = 1450 \times 2$

$2d_2 = 2900\ m$

Depth of the sea $d_2 = \frac{2900}{2} = 1450m$

Difference $\Delta d = d_1 - d_2$

$\Delta d = 2537.5 - 1450$

$$\boxed{\Delta d = 1087.5\ m}$$

4. Given:

Wavelength $\lambda = 50m$

Formula used:

Path difference $= (2n-1)\frac{\lambda}{2}$

For minimum n = 1

$AB = 2R$

Path difference $= \pi R - 2R$

$\pi R - 2R = \frac{\lambda}{2}$

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$R(\pi\text{-}2) = \frac{\lambda}{2}$

Radius of the semi-circle, $R = \frac{\lambda}{2(\pi-2)}$

Solution:

Radius of the semi-circle, $R = \frac{\lambda}{2(\pi-2)}$

$R = \frac{\lambda}{2(\pi-2)}$

$R = \frac{50}{2(3.14-2)}$

$R = \frac{50}{2(1.14)}$

$R = \frac{50}{2.28}$

$R = 21.875\ m$

$\boxed{R = 21.9\ m}$

5. Solution:

In an octane, let the frequencies be f, (f+n), (f+2n),.....,(f+(N-7))n

Sum of two frequencies = 2f

$f + (N-1)n = 2f$

$(N-1)n = 2f - f$

$\boxed{f = n(N-1)}$

6. Given:

Amplitude of sound waves is $a = 2a_o$

Frequency of sound wave is $f = \frac{f_o}{4}$

Formula used:

$I \alpha a^2 4^2$

Solution:

$I_{new} \alpha a^2 f^2$

$I_{old} \alpha a_o^2 f_o^2$

$I_{new} \alpha (2a_o)^2 \left[\frac{f_o}{4}\right]^2$

$I_{new} \alpha 4a_o{}^2 \frac{f_o^2}{16}$

$I_{new} \alpha \frac{a_o^2 f_o^2}{4}$

$\boxed{I_{new} \alpha \frac{I_{old}}{4}}$

7. Solution:

For a closed pipe $l = \frac{\lambda}{4}$

$\lambda = 4l$

For an open pipe $l = \frac{\lambda}{2}$

$\lambda = 2l$

Fundamental frequency of a closed pipe

$f_c = 250Hz$

$f_c = \frac{v}{\lambda}$

$f_c = \frac{v}{2l}$

Fundamental frequency of an open pipe

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$f_o = 2(\frac{v}{4l})$
$f_o = 2\, f_c$
$f_o = 2(250)$
$f_o = 500Hz$

Frequency of open organ pipe = 500Hz

8. Given:
Velocity of sound = 330 m/s
Velocity of police car v_s = 20m/s
Frequency of sound wave of the police car v = 300Hz
Frequency of stationary siren = 400Hz
Solution:
Let the speed of thief be v_o
Frequency of police siren heard by the thief

$f_1 = \left[\frac{300-v_o}{330-20}\right] 300$
$f_1 = \frac{(300-v_o)300}{310}$

Frequency of stationary siren heard by thief

$f_2 = \frac{(330+v_o)400}{330}$

If there are no beats, then $f_1 = f_2$

$\frac{(300-v_o)300}{310} = \frac{(330+v_o)400}{330}$
$(300 - v_o) \times 0.9677 = (330 + v_o) \times 1.2121$
$v_o = \frac{0.2444 \times 330}{2.1798}$

v_o = 36.99 m or 37m

9. Given:
Function a → $y = x^2 + 2\alpha tx$
Function b → $y = (x + vt)^2$
Formula used:
For the function to be a wave function.

$\frac{\frac{dy}{dx}}{\frac{dy}{dt}}$ *must be a constant*

Solution:
For function a
$y = x^2 + 2\alpha tx$
$\frac{dy}{dx} = 2x + 2\alpha t$ → *(1)*
$\frac{dy}{dt} = 2\alpha x$ → *(2)*
Dividing equation (1) by (2), we get
$\frac{\frac{dy}{dx}}{\frac{dy}{dt}} = \frac{2x + 2\alpha t}{2\alpha x}$ *is not a constant*

Function a is not describing wave

For function b
$y = (x + vt)^2$
$\frac{dy}{dx} = 2(x + vt)$ → *(3)*
$\frac{dy}{dt} = 2(x + vt)\, v$

$\frac{dy}{dt}$ = *2v(x + vt)* → *(4)*

Dividing equation (3) by (4), we get

$\frac{\frac{dy}{dx}}{\frac{dy}{dt}} = \frac{2(x+vt)}{2v(x+vt)} = \frac{1}{v}$ = *constant*

Function b is a wave function

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